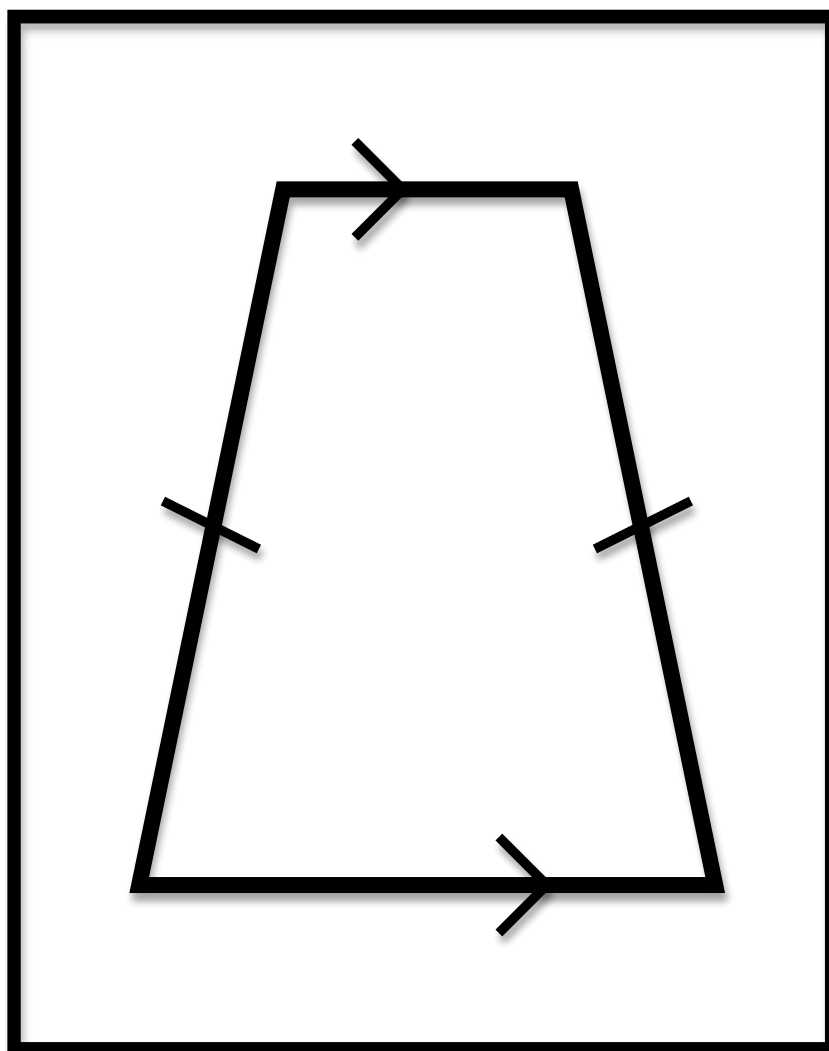


vinculum

Journal of the Saskatchewan Mathematics Teachers' Society

Volume 2, Number 1 (April 2010)

CURRICULAR EDITION



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SMTS objectives – as outlined in the January 1979 SMTS Newsletter – include:

1. To improve practice in mathematics by increasing members' knowledge and understanding.
2. To act as a clearinghouse for ideas and as a source of information of trends and new ideas.
3. To furnish recommendations and advice to the STF executive and to its committees on matters affecting mathematics.

vinculum's main objective is to provide a venue for SMTS objectives, as mentioned above, to be met. Given the wide range of parties interested in the teaching and learning of mathematics, we invite submissions for consideration from *any persons interested in the teaching and learning of mathematics*. However, and as always, we encourage Saskatchewan's teachers of mathematics as our main contributors. *vinculum*, which is published twice a year (in April and October) by the Saskatchewan Teachers' Federation, accepts both full-length **Articles** and (a wide range of) shorter **Conversations**. Contributions must be submitted to egan.chernoff@usask.ca by March 1 and September 1 for inclusion in the April and October issues, respectively.

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EDITORIAL: CURRICULAR EDITION

Egan Chernoff

I want to take this opportunity to talk about a war being fought – this very moment – on US soil. Don't worry; the Saskatchewan Council of Social Sciences (SCSS) (another of the Special Subject Councils of the Saskatchewan Teachers' Federation) has not hijacked this editorial! Here's the proof: I am not going to talk about the War of 1812, the Civil War, or the American Revolutionary War; but I am going to talk about the Math Wars.

The *Math Wars* is a term used to describe a mathematics education (i.e., research and practice associated with the teaching and learning of mathematics) debate over mathematics curricula, which began, and still goes on today, in the United States. "The immediate origins of the conflicts can be traced to the 'reform' stimulated by the National Council of Teachers of Mathematics' [NCTM] *Curriculum and Evaluation Standards for School Mathematics*" (Schoenfeld, 2004, p. 253). The NCTM's (1989) standards document not only acted as the Math Wars powder keg, it also helped define the two sides of the debate: *traditional* mathematics (curricula) versus *reform* mathematics (curricula).

On the one hand, proponents of the traditional approach, denoted by Schoenfeld (2004) as *traditionalists* value (note: I am aware I am painting with a broad brush here): skills, procedural understanding, repetition, memorization, algorithms, and direct instruction. In fact, the colloquial phase often associated with traditional mathematics (curricula) is *back-to-basics*. Whereas, and on the other hand, proponents of the reform approach, denoted by Schoenfeld as *reformers*, value (note: I am still painting with broad strokes): discovery, process, problem solving, inquiry, non-standard algorithms, creativity, pattern recognition, group work, conceptual understanding, communication, and alternative instructional methods.

You may be wondering what all this has to do with teaching mathematics in Canada. Since 1989 the NCTM's influence on mathematics curricula has moved out of the United States, into North America, and beyond. Case in point: The NCTM (2000) published the *Principles and Standards for School Mathematics*, which (in my opinion) has heavily influenced the Western and Northern Canadian Protocol's (WNCP) *Common Curriculum Framework (CCF) for K-9 Mathematics* (2006) and *Common Curriculum Framework for 10-12 Mathematics* (2008). For example, problem solving, communication, connections, and reasoning concurrently exist as *mathematical processes and standards for school mathematics* (which are central components both) for the WNCP and NCTM, respectively.

Now you may be wondering what this has to do with teaching mathematics in Saskatchewan. As I have detailed in the past, the mathematics curricula in the province of Saskatchewan is in the midst of major change, and will be for the next number of years (see, for example: <http://www.education.gov.sk.ca/math-curricula>). I have also contended that the new curricula, arguably, introduces a new approach to teaching and learning mathematics. Here's why: If the principles and standards of NCTM are influencing the CCF of the WNCP, which influences the Saskatchewan mathematics curricula, it can be argued, syllogistically if arranged properly, that the NCTM is influencing the Saskatchewan mathematics curricula. But, I don't want to bury the lead too much.

As I detailed above, the NCTM represents the reform mathematics (curricula), side of the Math Wars. I have also argued, above, that the NCTM is influencing Saskatchewan mathematics curricula. As such, the new Saskatchewan mathematics curricula are implicitly aligned with reform mathematics curricula. Now, let me ask you a question: Are you a traditionalist or a reformer? Either way, your response won't have too much impact on the new mathematics curricula in our province.

That said, your response will have a HUGE impact on the implementation of the new mathematics curricula in our province.

If you are a reformer, this new curricula is right up your alley. If you are a traditionalist or just aren't sure which side of the debate you are on yet, I ask of you two things. First, be aware of the issues associated with implementing reform curricula in a traditional manner. Second, remember that the traditional versus reform debate is a false dichotomy and use this fact to your advantage. Whether you are a reformer, a traditionalist, or some where in between I hope this issue of *vinculum* and our upcoming SUM conference can help you with the implementation of our new mathematics curricula here in Saskatchewan

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PRESIDENT'S POINT

Stephen Vincent

It is similar to reading a problem and wondering which way you should proceed. Several paths are possible, but which one is best given the present situation. In the same way, the SMTS is at a crossroad. SMTS is currently involved in planning and hosting two conferences: Sciematics and SUM. As a reader of this journal, there is a high probability that you have attended one or both of these conferences over the last two years. There is also a high probability that you have heard about the uncertainty of the

SMTS continuing in partnership with the Saskatchewan Science Teachers' Society (SSTS) beyond Sciematics 2010. There are two main reasons for this uncertainty.

First, following Sciematics 2008, members of the SMTS executive wanted, as a goal, to further meet the needs of math educators at all levels. The purpose of SMTS, as stated in our constitution, is to "foster excellence in mathematics education in elementary, middle, secondary, and post-secondary education institutions" (Section 3.1). In order to meet this goal for math teachers of all levels, members of the executive planned a conference for teachers in May of 2009. Due to the direct focus on math education, the conference was able to feature three internationally recognized leaders in math education as well as a number of quality working group sessions. The response from the conference attendants was very positive, encouraging the SMTS to host the SUM Conference again this year.

The second reason for the uncertainty of Sciematics is the shortage of volunteers on the planning committee. These positions are by no means limited to people on the SMTS executive, but are open to the general membership. In the last four years, however, the SMTS has not provided sufficient help in the organizing and planning of Sciematics. In being fair to the SSTS, we cannot continue with this trend. We either need to step-up our involvement or step-out of our partnership with SSTS.

In order to deal with the question of Sciematics and SUM in a timely manner, the SMTS is having its biannual general meeting on May 7th during the SUM Conference at the U of S. We encourage all of our members to come to this meeting and contribute to the future direction of the SMTS. There are several ways in which we can proceed concerning Sciematics and SUM. As an executive and membership body, we want to ensure that we are creating the best possible opportunities to foster excellence in mathematics education for all teachers in our province.

WRITING TO COMMUNICATE IN THE SECONDARY MATHEMATICS CLASSROOM

Nathan Banting

There is a popular school adage that mnemonically recites the three essentials to an effective school experience: reading, writing, and arithmetic. For many years educators have compartmentalized these “Three R’s” into respective content areas; arithmetic skills are gained in a mathematics classroom, while writing and reading are developed in the linguistic orientated subjects like english and history. It was not until 1989 – with the *Curriculum and Evaluation Standards for School Mathematics* document – that the National Council of Teachers of Mathematics (NCTM) documented that communication is an important goal of mathematics education, and that this goal is best accomplished when students “have an opportunity to read, write, and discuss ideas” (NCTM, 1989, p. 8) in mathematics class as well as in english and history classes. The standard of communication relies so heavily on language that Vazquez (2008) considers it “as important to learning mathematics as it is to learning to read” (p. 19). The NCTM makes it clear that students are to move away from archaic forms of drill and practice and move toward expressing mathematical ideas orally and in writing (NCTM, 1989). The newly stressed communication focus can take three forms in the classroom: an inner communication or metacognition, a communication between peers in the classroom, and clearer communication between the learners and the teacher. Implementing writing in the mathematics classroom leads to increased metacognition for students, improved class discussion, and

explanation of, mathematical strategies, and an authentic assessment tool for teachers.

The first form of writing is the development of metacognition. This skill is not easily developed in any high school classroom as Schloemer (1993) discovered when she experimented with NCTM guidelines in her secondary math class. It was no surprise that she discovered it took a great deal of practice for students to write what they are thinking, because they have always been taught that the correct answer is the ultimate goal. The NCTM’s communication standard reverses this goal and elicits student discovery of the “how” and “why” of every problem. Students need to become aware of why they apply certain algorithms and formulae if they are to utilize the knowledge presented to them in the classroom. Writing activities not only create this student awareness; the act of writing also develops new mathematical thought—a process coined Writing-to-Learn. Zinsser (1988) observed this phenomenon in mathematics students; as they began to scribe their current knowledge on a subject, “their thinking kept changing as they wrote and discovered what they really thought...writing and thinking and learning had merged into one process” (pp. 165-166). Schloemer (1993) found that simple writing assignments helped students realize that they did not understand a topic. Writing activities in the math classroom articulate old thoughts, generate new thinking and learning, and eliminate ‘mindless math’. Writing focuses students on re-evaluating and communicating their thoughts in the form of coherent mathematical thought.

The second form of communication that writing develops is communication between peers in the form of class discussion. Students are expected not only to present the answer and strategy they used, but also “analyze, compare, and contrast the meaningfulness, efficiency, and elegance of a variety of strategies” (NCTM, 2000, p. 268). This goal works directly from the development of metacognition. Students cannot compare their respective strategies until they can articulate them introspectively. The essence of this communication lies in the students’ ability to “connect the inner dialogue with the outer dialogue” (Anson & Beach, 1995, p. 40). Writing assignments develop personal strategies for individual students. After this has taken place, effective strategies are presented and contrasted. Writing creates an environment where all students are afforded a comfortable place to express their mathematical thought (Crawley, 2009). The increased involvement strengthens class discussions because it gives every student the opportunity to reflect on their thoughts before sharing them with the class (Dougherty, 1996). Students begin to express what it is they find effective or eloquent. Jurdak and Zein (1998) concluded that writing in mathematics increased procedural knowledge, conceptual understanding, and mathematical communication. These three capacities will naturally increase a student’s ability to understand mathematical concepts and procedures, and, therefore, carry on discussions to compare methods that are increasingly efficient. Writing in the mathematics classroom develops a critical eye for the procedure used to attain an answer. The emphasis is taken away from the

solution, and placed on processes for attaining the correct answer.

The third form of communication, developed through writing, is that between teachers and students. Assessment in mathematics is largely focused around attaining answers by applying a set of rules, but questions that “focus on applying an algorithm cannot assess how students connect mathematical ideas” (Dougherty, 1996, p. 556). Effective communication between student and teacher creates a more authentic assessment experience. Infusing writing with content will help teachers identify when students are developing misconceptions (McIntosh, 1991). A student’s answer may be correct, but the logic or procedure behind it may still be faulty. When communication becomes the center of assessment, assessment moves from an intermittent format of samples to a framework that is continuous and dynamic (Countryman, 1992). Writing can reveal future problems before they occur; there is an added element of authenticity in assessment when writing is involved because writing opens a window into the student’s thoughts, and shows the process of attempting to solve the problem (Vazquez, 2008). Vazquez insists that unless she could tell how a student arrived at a solution, even a correct one, she had little chance of truly assessing whether or not the student understood the problem. Assessment is also re-routed to focus on communication. Thus, a correct answer, although an important goal, is no longer a true indicator of correct understanding.

According to the NCTM, communication should be a forefront feature in all mathematics classrooms. Writing in the classroom more accurately represents student

understanding. It can be used to discern misconceptions before they grow into major confusion, and also to gather an authentic representation of student growth and learning.

Communication is a vital component of an effective mathematics experience. Writing assignments allow students to develop critical communication skills internally as metacognition, and externally between peers and teachers alike. The intrapersonal communication results in increased external communication between peers, and eventually a continuous and dynamic assessment tool for teachers. Communication becomes a tool to restate metacognitive discoveries as well as discover others that may be more effective for particular students. Writing is a vital component of mathematical communication, and should not be segregated from the discipline at any level – especially the secondary level. All three R's should be effectively reunited in the mathematics classroom.

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TO BECOME WISE TO THE WORLD AROUND US: MULTIPLE PERSPECTIVES OF RELATING INDIGENOUS KNOWLEDGES AND MATHEMATICS EDUCATION

Gladys Sterenberg, Liz Barrett, Narcisse Blood, Florence Glanfield, Lisa Lunney Borden, Theresa McDonnell, Cynthia Nicol, and Harley Weston

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Introduction

Significant curricular initiatives in mathematics have been undertaken across Canada to appropriately and respectfully consider Indigenous knowledges and perspectives. For example, the Common Curriculum Framework for K-9 Mathematics: Western and Northern Canadian Protocol (Alberta Education, 2006) now describes Aboriginal learners in its Program of Studies:

Aboriginal students in northern and western Canada come from diverse geographic areas with varied cultural and linguistic backgrounds. Students attend schools in a variety of settings including urban, rural and isolated communities. Teachers need to understand the diversity of cultures and experiences of students.

Aboriginal students often have a whole-world view of the environment in which they live and learn best in a holistic way. This means that students look for connections in learning and learn best when mathematics is contextualized and not taught as

discrete components.

Aboriginal students come from cultures where learning takes place through active participation. Traditionally, little emphasis was placed upon the written word. Oral communication along with practical applications and experiences are important to student learning and understanding. It is also vital that teachers understand and respond to non-verbal cues so that student learning and mathematical understanding are optimized.

A variety of teaching and assessment strategies is required to build upon the diverse knowledge, cultures, communication styles, skills, attitudes, experiences and learning styles of students. (p. 3)

These initiatives, which call on teacher educators, teachers, administrators, and students to reconsider their received understandings of Indigeness, and to challenge inherited conceptual frames derived from the lengthy colonial process of looking for mathematics in cultural activities and by doing so validating the activities as mathematical from a Euro-Western perspective. We wonder about possibilities of enacting an Aboriginal perspective within our educational contexts.

In May 2009, a group of mathematicians and mathematics educators of non-Aboriginal and Aboriginal descent participated in a study group on Indigenous knowledges at the Canadian Mathematics Education Forum. For three days, we engaged in authentic conversations (Clark, 2001) as we investigated the following questions: What do Indigenous knowledges offer for teaching and learning mathematics

by both Aboriginal and non-Aboriginal students and teachers? What role does language play? What role does place, community, and culture play? Narcisse Blood, recognized as an *litsitsskopa* (emplaced-for-a-reason, or elder), led our conversations and offered guidance and wisdom in our endeavour to better understand the relationship between Indigenous knowledges and teaching mathematics.

This paper continues these conversations as we draw on our educational experiences to consider the following question: What is the relationship between Indigenous knowledges and mathematics education in our current research/teaching projects? We have organized stories of our multiple perspectives into four sections: Culturally responsive education, Language and culture, Learning mathematics from place, and Relationships – responsibility – recursion. Our hope is that you may be inspired to appropriately and respectfully consider Indigenous knowledges and perspectives in your teaching and learning.

Culturally Responsive Education

Narcisse Blood

What has culture come to mean today? Culture came from anthropologists and the old country. The view of people back then was that any society other than ours is inferior. And anthropologists started saying, "No, that's not the case. As a matter of fact, some societies are every bit as complex if not more complex than us." That's where the term 'culture' started from. But today, it's changed. It's not that. It has a different connotation, especially where I work and in the community.

Culture has evolved into a definition of right. We're going to work with the [Universities] and they say, "Oh yeah, let's not leave out the cultural component." You know, as an after thought. "Let's get Ryan and Narcisse in there to appease." It sounds good. Because when I was in high school, Indian Affairs came up with that same kind of thing: "We'll throw in culture. We're going to teach them how to bead." And they brought Mrs. Rosie Day Rider and the late Louise Cropped Ear. They knew what they were up to. In the classes, you'd be doing beadwork, and they'd be telling stories. It was not what Indian affairs wanted, you know, "We'll throw a bead and feather in there to shut them up."

Cynthia Nicol

For the past few years I have had the honour of working with researchers at the University of British Columbia along with teachers, students and community members in two rural communities and one urban community. In each community we are exploring what it means to live culturally responsive mathematics education. Our work is guided and informed by Indigenous knowledges in multiple ways. First our project brings together both Aboriginal and non-Aboriginal scholars, school-based teachers and researchers, students and community members to re-imagine mathematics teaching and learning. Second our work is informed by and guided by the framework of Indigenous storywork developed by co-researcher Joann Archibald. Working with teachers we are exploring the seven principles of reverence, respect, reciprocity, responsibility, synergy, interrelatedness and wholism in terms of our research and mathematics pedagogy.

We are exploring our awareness of these principles, how they can be used to connect mathematics, community, culture and Indigenous knowledges and what this could mean for improving students' mathematical experiences and emotions. For example, Indigenous storywork as a methodology focuses our attention on the importance of building relationships that are respectful and mindful of community protocols, of deeply appreciating the stories and experiences shared, of giving back and paying forward our work so that future teachers and students may learn from our experiences, and considering our multiple responsibilities in the research process. Experiences with the land and nature emphasize the value of and connection of the inner passion, feel, and heart with the environment. For us, Indigenous epistemologies can be characterized as being experiential, storied, relational, contextual and wholistic.

Our goal is transforming mathematics education so that it serves the diverse interests and aspirations of all Aboriginal and non-Aboriginal students within our project communities and beyond. Our goal is actualized by the following processes: 1) embracing a wholistic and interconnected view of mathematical knowledge; 2) exploring practical approaches for using local and traditional knowledges as resources for mathematics pedagogy; and 3) using local cultural values to extend possibilities for pedagogical practices, connecting with students, and learning from students.

Project teachers are exploring culturally responsive mathematics education in multiple ways. One is by developing mathematics problems inspired by traditional legends. For

example, teachers designed a lesson from an audio recording of *Raven Brings the Light* as told by community elders and youth. In this story the Raven brings light to the world after taking it from a series of nested bentwood boxes. The lessons teachers designed and are currently piloting in their classrooms invite students to listen to the story and then explore the mathematics of building bentwood boxes out of paper.

A second way of exploring culturally responsive education is through developing digital storybooks. Teachers and students have collected a series of images of the land and activities connected to the land. Together they have designed three mathematics photobooks that involved developing, adapting and writing mathematics problems inspired by the images.

A third context for exploring culturally responsive education has involved collaboration with the local museum. In this context teachers are exploring the mathematics of the process of canoe building from sapling to sea. Teachers are researching their own experiences of culturally responsive education within their classrooms and are meeting once every couple months to share their ideas and stories.

Harley Weston

During the past three years I have had the opportunity to develop a relationship with a number of Aboriginal Elders and other leaders in the Aboriginal community in Western Canada. I feel extremely privileged to have benefited from these leaders sharing their knowledge. Their knowledge has guided Judi MacDonald and me in the development of resources for use by teachers, students and parents in the Aboriginal and non-Aboriginal

communities. These resources, which are being written by education students at the University of Regina and the Gabriel Dumont Institute, grow out of the audio and video capture of activities and conversations with people in the Aboriginal community. The use of audio and video material in this initiative facilitates the inclusion of a cultural context as an integral part of the resources as well as making the content accessible to students in younger grades.

The following are three examples of the activities/conversations for which teaching resources are being prepared.

- Elder Glen Anaquod from the Piapot First Nation led the students from Kitchener School through a tipi raising. This is an inner school in Regina where many of the students are Aboriginal and through this activity Elder Anaquod shared traditional Saulteaux teachings with the students.
- Birch bark biting is an art form produced by folding a paper-thin sheet of birch bark and placing a design on it through perforating the birch bark by biting. The artist, Rosella Carney is a Cree woman from La Ronge Saskatchewan. Rosella is also a Cree language instructor and her interview includes a discussion of the number words in Cree.
- Cassandra Opikokew was a student in the Indian Communications Arts program at the First Nations University of Canada and the School of Journalism at University of Regina from which she graduated in 2009. In her interview Cassandra talks about how she uses the skills she gained in both programs and the importance of Aboriginal issues and voices in the media.

Language and Mathematics

Narcisse Blood

What we're talking about is a language that was built over thousands of years. So one of the barriers to learning is also time, concepts of time, constructs of time. We work with the museums or some of the parks like Writing on Stone Provincial Park. We have something similar about preserving sacred sites, but their concept of preserving is it's in the past. By virtue of us being there and speaking the language and looking at the petroglyphs, takes it out of that model and brings it to the present. And to get that across is quite difficult but there's movement along that line. And therefore, when we open a bundle, it's, if you want to put it this way, it's history happening right there. So the language is that, it's evolved over thousands of years.

Lisa Lunney Borden

I began my teaching career in We'koqma'q First Nation on Cape Breton Island in Nova Scotia, which is a Mi'kmaw community. Mi'kmaw communities in Nova Scotia have a unique jurisdictional agreement with the Government of Canada that gives them control over their education system and collective bargaining power. These schools live within the tension of wanting to provide culturally-responsive and language-rich programs for their students while at the same time being required by law to offer provincially approved curricula.

Disengagement from mathematics and science is a concern for many teachers in these schools as they grapple with the tensions between school-based mathematics and Mi'kmaw ways of reasoning about things seen as

mathematical. Having taught secondary mathematics for ten years I had experienced these tensions myself. It was this experience, and the related learning from community members, that brought me to my doctoral work. My goal was to work towards the development of culturally responsive mathematics curricula with participant schools, yet it soon became evident that a necessary first step was to investigate the tensions between Mi'kmaw cultural ways of knowing and school-based mathematics.

My research took place in two schools and involved after-school discussion with teachers in the form of *mawikinutimatimk* (coming together to learn together). This traditional community practice values the contributions of all participants and acknowledges that we each have something to teach and something to learn. Through our conversations, four key areas of concern emerged as themes: 1) the need to learn from Mi'kmaw language; 2) the importance of attending to value differences between Mi'kmaw concepts of mathematics and school-based mathematics; 3) the importance of attending to ways of learning and knowing; and 4) the significance of making ethnomathematical connections for students. Although interconnected and interdependent, each of these themes can be tied to the need to learn from Mi'kmaw language.

The important role of indigenous language in understanding mathematics was demonstrated by Denny (1981) who used a "learning from language" approach while working with a group of Inuit elders in Northern Canada to explore mathematical words in the Inuktitut language. Rather than developing curriculum and translating it

into Inuktitut, they used the mathematical words to develop the curriculum and associated mathematics activities. More recently Bill Barton (2008) has shared the stories of his similar struggles in translating mathematics concepts into the Maori language. He has argued that mathematics evolves with language and as such claims that:

A proper understanding of the link between language and mathematics may be the key to finally throwing off the shadow of imperialism and colonisation that continues to haunt education for indigenous groups in a modern world of international languages and global curricula. (p. 9)

This connection between language and decolonization of education was certainly evident in the conversations with participants during my doctoral work.

The principal of one of the participant schools arrived at our session one day and told me that I should call my dissertation "lost in translation." She proceeded to talk about the difference between *Inuitasi* (Our people's ways of thinking) and *aklasiweitasi* (Anglophone ways of thinking). She argued that many conflicts arise for children when their ways of thinking (*Inuitasi*) come into conflict with teachers who have different ways of thinking (*aklasiweitasi*). There is something being lost in the translation of worldviews, of ways of thinking and styles of communication. She claimed that she sees the conflicts arising on a daily basis as the students in her school struggle to find their way through a colonizing curriculum. Something is being lost in translation and she is searching for ways to resolve this

challenge. The Mi'kmaw language holds the key to *Inuitasi*.

Three key pieces emerged in our discussion about the role that language plays in understanding the link between Indigenous knowledge and mathematics. First, there was an expressed need to include more Mi'kmaw language in the mathematics classroom. This group in particular stressed the importance of reclaiming mathematical words and supporting Mi'kmaw speaking teachers to develop a lexicon of words that could be used in their classes. Many participants shared their belief that using the Mi'kmaw language as much as possible can only benefit students. Several participants expressed how much more responsive students were when they were asked to complete a task or do more work in Mi'kmaq. In particular, one teacher participant commented on the way in which her students often don't understand what she means when she says 'how many' but noted, "say '*Tasikl* (how many – inanimate)?' and they get it."

Second, there is value in asking questions such as "What's the word for...?" or "Is there a word for...?" through raising such questions, our groups began to gain new insight into the ways of thinking (*Inuitasi*) embedded in the language. It was often most interesting to discover that a certain word that was taken-for-granted to be understood by children in school-based mathematics perhaps did not even exist in the Mi'kmaw language. 'Flat' is one such word. There is no word for 'flat' in Mi'kmaw yet it is commonly used in mathematics. When I asked elders in the community how they would describe the bottom of the basket which is flat (based on an Anglophone worldview) I was told it was "just the bottom of the basket. It's

what lets it sit still." And when I asked about a flat tire I was told that it was "out of air." If 'flat' is not a word that is used in Mi'kmaq, what happens when this word is used in mathematics classrooms and it is often taken-for-granted that there is a shared understanding of its meaning?

Third, there is a sense of motion embedded in the Mi'kmaw language that is not apparent in school-based mathematics. Shape and space words act as verbs in Mi'kmaw and are dynamic. For example, to describe something that is straight, Mi'kmaw speakers might say "*pektaqtek*" which means "from here to there it pretty much goes straight." This dynamic nature of mathematical ideas in Mi'kmaq comes into conflict with the tendency in school mathematics towards nominalisation – turning actions and processes into nouns. Our group wondered what might happen if we drew on the verb-based discourse of Mi'kmaq rather than the noun-based discourse of school mathematics to engage our Mi'kmaw students in the learning of mathematics. Such 'verbification' – utilizing the verb-based discourse – may provide Mi'kmaw students with increased understanding of and connection to mathematical concepts (see Lunney Borden (2009) for a more detailed discussion).

While there is much more work to be done to explore the connections between Aboriginal languages and Indigenous knowledge for mathematics education, this certainly seems to be an important area of investigation to support the development of culturally responsive mathematics education.

Liz Barrett

Something truly unique has happened in Mount Currie, British Columbia.

Xit'olacw Community school is proud to be one of the oldest Band operated schools in Canada and a group of dedicated elders and parents have worked tirelessly to ensure that it is one of the leading schools in First Nation Language Education. Reclaiming and protecting the language of the community is a high priority of the school. They have one of the few immersion programs here, so students can learn and understand Lilwat, St'at'imcets which is the Lilloet language.

The teacher at the Clao7alcw (Raven) School, Terri Williams invited me in to her class so that I could model some math strategies that are very effective with teaching young children. The idea struck me that we should teach the same math program being used by other classes at the school, but we should translate this into the language. Everyone was excited by the idea and so we contacted the author of the math program, who was very supportive of the idea and gave us permission to start the translation of the Grade 1 books.

This was the start of a language and math adventure. Funding was received from *First Voices* and so Lois Joseph, team leader of the Lil'wat7ul Culture Centre used the funding to get Elders together to start the intense discussions as to what language would be best used to describe the mathematical concepts that were in the books. New language had to be created to describe things like squares and rectangles, triangles and core patterns and so the process was a collaborative living process of embracing and agreeing on new words that could and should be used.

What was unique for me was watching the process unfold and

observing the respect that the Elders showed to each other throughout the discussions. This was and is still a mammoth team effort and I wish to pay tribute to the team made up of Mary James, Laverne James, Priscilla Ritchie, Georgina Nelson, Veronica Bikadi, Jean Andrew, Vera Edmunds, Theresa Jones and Dixie Joe. Tanis Grandbois helped to capture the data electronically and Burt Williams is helping to edit the text. We thank Lois Joseph for all her support of the project and we look forward to other communities following the lead of Mount Currie.

Nitawahsin-nanni: Learning mathematics from place

Narcisse Blood

[My colleague and I] were asked to come and share [at the University]. But what stayed with me is, here are the buildings and there's a parking lot. And here are trees that go down to that environment. What struck me is the dissonance in terms of learning. What transformation will take place for the University to acknowledge the place and how tragic that can be. Because we're sitting up there overlooking the harbour. And you have all these ships and all these raw materials that are going to be shipped away somewhere. And how unsustainable that is.

Theresa McDonnell

Infusing the curriculum with Aboriginal perspectives is what Alberta teachers are now required to do. Over the last two years, I have taken on this challenging but important endeavour in my middle school mathematics classes. I teach in Siksika, Alberta but myself am of Cree and Irish-Canadian decent. Siksika is one of the member nations of the Blackfoot Confederacy.

In my classroom, I have observed silent struggles with identity creating the greatest barriers to success among my students. They long deeply to learn more of their proud history and accumulated knowledge while being profoundly involved and invested in a global world.

In this setting, my experience has taught me that teaching from an Aboriginal perspective is best accomplished by focusing on the perspectives of the Siksika people. As their knowledge has accumulated over the millennia in one place, it is fair to suggest that Blackfoot perspectives arose from their interaction with the land now labelled Southern and Central Alberta, Eastern Saskatchewan, and Northern Montana. This knowledge is rich in mathematics. I came to see my job as a teacher as empowering Siksika students in recognizing their intuitive mathematical abilities. This was partly done by making time in the schedule for learning from place.

Learning from place continues to be a valid and meaningful method of interpreting and understanding the world, including mathematics. As most mathematicians know, mathematics can be found everywhere and nature is no exception. Therefore, I decided not to rely solely on mathematical textbooks as reference and guide. Instead we explored mathematics in the world as well as the textbook. They complimented each other well. For example, I taught part of both the Shape and Space units using the spokes, angles and circular geometry of the Majorville medicine wheel site. Students learned concepts with help from the textbook, but moreover, concepts were explored in activities at the site, and in the creation of models afterward in the classroom.

By taking students to sacred sites within the Blackfoot Confederacy and bringing some lessons outside, we soon realized that the land became a very present third party. Lessons don't follow the same schedule that a teacher carefully plans when taught outside. Suddenly other factors arise such as weather, animals and other unexpected guests, as well as an abundance of student curiosity. This alternative method of teaching mathematics is challenging because it requires a plethora of planning, a field trip budget, awareness of protocols, support from culturally knowledgeable staff or community members, flexible and supportive administration, and a willingness to relinquish some power and structure in teaching.

How each teacher accomplishes meeting the new goals set out by the province of Alberta will be varied and unique. For me, "infusion" is the wrong term. Teaching from an Aboriginal perspective is simply finding what is meaningful and relevant to the students, that honours the ancestors of the host territory in which teachers live and teach. It means teaching the curriculum and addressing silent identity issues simultaneously by revering the land and people from which the students came. This can be accomplished by continuing to find meaning in places and inviting students to see the world mathematically and intuitively.

Gladys Sterenberg

Working alongside my Aboriginal colleagues and friends, I am coming to understand my unique role as a person who can provide bridging experiences among teachers of federally – and provincially – funded schools. Perhaps this is best illustrated by a story of one such experience.

In my work as a mathematics teacher educator, I have been invited to work in partnership with teachers in several First Nation communities. At one First Nation high school, we are investigating how learning from place might be enacted. To date, we have taken the students on two field trips to sacred Blackfoot sites, one of which was to the Big Rock. Located west of Okotoks, Alberta, the Big Rock is the largest glacial erratic in the world and is part of a series of boulders stretching from Utah to Jasper. This site is significant to the Blackfoot community and the splitting of the Big Rock by Napi's – a Blackfoot trickster – actions is a familiar story. On our field trip to the Big Rock, the elder who was with us began our visit with an offering and stories of Napi. While we were there, elementary students from the Big Rock School in Okotoks arrived to investigate the meaning of their school name. I knew one of the teachers of this group and once I found out why they were there, I suggested that perhaps her students might want to hear the Napi stories told by the elder. Following protocol, I asked the elder if he could share the stories with them and he was very excited about the opportunity to do this. The elementary students were highly respectful when listening to him. The principal of Big Rock School wrote an affirming editorial response about this experience and this was published in a local newspaper. In this particular situation, my comfort with federally – and provincially – funded school communities facilitated a bridging experience. For me, this was a profound experience as I better understood the importance of creating opportunities for sharing knowledge between Aboriginal and non-Aboriginal communities and the

possible role I might have in this process.

For me, relating Indigenous knowledges and mathematics education is focused on the act of listening. As a non-Aboriginal visitor, I am respectful of protocols, always mindful of my place within this place. Learning from place recognizes the intimate relationship that Indigenous people have with the land. In our visits to sacred places, the land has spoken to me as I have listened. Stories of place have grounded our visits and our response to knowing the land has informed our acts. Carbaugh (1999) describes listening as dwelling-in-place. He suggests that Blackfeet listening is a “highly reflective and revelatory mode of communication that can open one to the mysteries of unity between the physical and spiritual, to the relationships between natural and human forms, and to the intimate links between places and persons” (p. 250). The sacred places have spoken to me as I have listened. Through the stories told to us by cultural Elders, I am learning to dwell-in-place.

Relationship – Responsibility – Recursion

Narcisse Blood

Really what it is for my people, and I think all of us and sometimes we forget, really what it is all about is this relationship. Ultimately, that's who we are. It's relationship. If you want to learn about the Blackfoot, it's about relationship and relationship and relationship. About everything. Those that we see, and those that we don't see. So with that, I'm starting to get to know a lot of you in here, it's good to see you. And that's important. In our world, we don't take that for granted.

Florence Glanfield

“Florence, ohmigosh this is FOIL! Why didn’t anyone ever tell me that there was a relationship? I knew how to multiply...but could never figure out FOIL!” These were the words of Cindy, a student in a mathematics education class for elementary preservice teachers, approximately 7 years ago. Cindy and her peers had been exploring the underlying concepts behind the multiplication of two two-digit numbers. In Cindy’s work with using base-10 blocks and the subsequent numerical representations, she came to realize that the multiplication of two two-digit numbers was the distributive property of multiplication, or in Cindy’s words, ‘FOIL.’ I now ‘hold’ Cindy’s words with me in my teaching and learning, and have ‘held’ Cindy’s words as I’ve come to know, and acknowledge, myself as an Aboriginal person in teaching mathematics.

When I consider Aboriginal perspectives in a mathematics classroom I think about the importance of Relationship. In my experiences as an Aboriginal person I’ve come to understand the importance of acknowledging relationships that we have in all aspects of my life. For example, people have relationships with the land (some call it Mother Earth) as human beings draw sustenance from the land and once we acknowledge the relationship with the land we then have a responsibility to sustaining the relationship and sustaining the land. A second example is that we have relationships with our family – we draw sustenance from our family and when we acknowledge that we draw sustenance from our relationship with family then we also have a responsibility for sustaining the relationships and

sustaining the family. Once we acknowledge the relationships that we have with the land and the family and we act on our responsibility, as being a part of that relationship then our initial relationship is now different from what it was. Hence we are in a continuous cycle of Acknowledging Relationship; Acting Responsibility; and Re-living or Re-telling the Relationship (Recursion).

What does this mean for a mathematics education class? Most of my teaching is now teaching pre-service teachers about what it means to teach mathematics. The focus of the planning work that I do in my classes is around this notion of Relationship – Responsibility – Recursion. In a classroom there are multiple perspectives on relationship that I believe a mathematics teacher ‘holds’ in their practices. To name a few – a teacher has a relationship with mathematics; with the experiences they had in learning mathematics; with colleagues in the school; with the community in which the school resides; with their worldview about what it means to learn, teach, and specifically teach and learn mathematics; with their family; with their students; and with themselves. I also believe that a teacher must acknowledge that within each of these relationships there are other relationships that can be named – for example within the relationship one has with mathematics one might be aware that there are relationships within the content of mathematics itself. My development as a mathematics teacher and mathematics teacher educator then becomes a life-long journey as I will continue to come to know about the multiple relationships that exist within my classroom and that I continue to have multiple responsibilities. In other words

as I attend to or notice (Mason, 2002) the complexity, and the responsibility, of the relationships that exist in my classroom then my teaching continues to focus on sustaining relationships; my life as a teacher is a series of recursive acts in sustaining relationships.

What might be an example? For example, suppose I am teaching about the multiplication of two binomials in a grade 9 or a grade 10 mathematics class. To focus on relationship, I might ask myself questions such as

- How is it that I understand the multiplication of two binomials?
- In what way(s) is the multiplication of two binomials related to multiplication of two two-digit numbers?
- How is it that I can invite my students to see the relationship between the multiplication of two binomials and the multiplication of two two-digit numbers?
- What were my experiences in learning about the multiplication of two binomials? How is it that those experiences have informed me in how I understand the multiplication of two binomials?
- How is it that learning the multiplication of two binomials contributes to my student's deeper understanding of mathematical ideas?
- In what way will my actions as a teacher in teaching about the multiplication of two binomials sustain my relationship with my students?
- In what way will my actions as a teacher in teaching about the multiplication of two binomials sustain the relationship my students are developing with mathematics?

- In what way does how I see myself as a learner of mathematics impact the decisions I make as a teacher of mathematics and the tasks that I ask my students to engage in when learning about the multiplication of two binomials?
- How is it that I share my understanding of the relationships within mathematics with my students in a way that sustains the relationship that I have with my students?
- How is it that I share my understanding of the relationships between the multiplication of two binomials and other areas of mathematics with my colleagues?
- How is it that I participate in the development of a shared understanding of these mathematical ideas within the community in which I teach?

The ideas that I write about here are a way that mathematics teachers might begin to focus on the relationships or the interconnectedness of our content, our lives as teachers, the lives of our students, and the context in which we teach. Bopp, Bopp, Brown and Lane (1988) write that, "the great lesson of the sacred circle is always that separate entities, when seen in light of the universe, are equal and necessary parts of the larger whole. It brings out the ancient teaching of the interconnectedness of all things" (p. 62).

Conclusion

Colonization was an intentional act. If we try to convince our students that the only mathematics that exists and is worth studying has its roots in Euro-Western traditions, then we are engaged in such an intentional act. By honouring multiple perspectives of mathematical thinking and knowing, we can come to

know mathematics and ourselves in a different way.

The *Common Curriculum Framework for K-9 Mathematics: Western and Northern Canadian Protocol* (2006) states, “The strategies used must go beyond the incidental inclusion of topics and objects unique to a culture or region, and strive to achieve higher levels of multicultural education” (p. 3). While we agree that token gestures of inclusion are inappropriate, we believe that relating Indigenous knowledges and mathematics education is context specific. Multiple perspectives are necessary and embraced. Moreover, any project must be community initiated.

At the end of our time together, we were left with many questions: How do we teach in ways that are responsive to students and challenging of the systemic inequities? How do we invite others to think with us in similar kinds of critical reflexive research? How do we engage in our own decolonizing? In this report, we offer our stories and challenge others and ourselves with our questions. As we reflect on our experiences within this working group, we contemplate the next steps to be taken and invite others to walk alongside us as we attempt to live our lives differently:

Take our stories. They’re yours. Do with them what you will. Use them in your planning. Tell them to other educators. Forget them. But don’t say in the years to come that you would have lived your life differently if only you had heard our stories. You’ve heard them now. (adapted from Thomas King, 2003)

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“PARENT ENGAGEMENT IN MATHEMATICS IS JUST NOT POSSIBLE.” OR IS IT?

Debbie Pushor

Recently, I was leading a session on parent engagement with a group of interns and their co-operating teachers. As I was presenting, establishing a sense of what parent engagement is, I was challenged by a grade 8 mathematics teacher who insisted, “Parent engagement in mathematics is just not possible. Parents do not know the curriculum,” he asserted. “And, further to that, they don’t know the math.” In regard to both statements, I believe his assertions have some validity. I think about myself as a parent. I have one son who has completed high school, and two who are currently in grade 10. While I have been to their Meet the Teacher and curriculum nights, attended their parent-teacher conferences, and monitored their homework throughout the years, could I name the curricular concepts in mathematics which are taught at each grade level? Could I articulate the curricular outcomes which are expected? The answer is no. Further, do I know or understand the mathematical concepts they are working with? Could I do the math my sons are doing? The answer is also no. While I did well in high school mathematics (thirty five years ago!), I chose a humanities pathway in my postsecondary education and in my teaching career. Other than the math that permeates my daily life, mathematics is not a central feature of what I do. And, as I realized, just the other day when my son Teague asked me about a homework question, I do not know anything about square roots of negative numbers! So, when I examine my knowledge of mathematics and mathematics curriculum, and I consider that my

experiences with them as a parent are probably fairly typical, must I conclude that the grade 8 mathematics teacher is right, “Parent engagement in mathematics is just not possible”?

Two key words are at play here: **teacher** and **parent**. What knowledge, skills and attitudes does a mathematics teacher need in order to teach mathematics? What is the teacher’s role and what are the expectations of the teacher in regard to that role? What knowledge, skills and attitudes does a parent need in order to be engaged in a child’s schooling in mathematics? What is the parent’s role and what are the expectations of the parent in regard to that role? I believe both teachers and parents are holders of “personal practical knowledge” (Clandinin, 1986; Clandinin & Connelly, 1995; Connelly & Clandinin, 1988, 1999) of children, and of teaching and learning. But, while I believe teachers’ and parents’ knowledge is complementary and equally important to the schooling of children, I do believe it is different knowledge.

Personal Practical Knowledge

In Connelly and Clandinin’s (1988) conceptualization of teachers’ personal practical knowledge, personal knowledge is the teacher’s knowledge gained from lived experience in all aspects of life – at work, at play, with family and friends, through hobbies and travel, and so on. Practical knowledge is the teacher’s knowledge of their contextualized setting – a classroom – gained through experiencing the dynamic interaction of persons, things and processes – their students, perhaps other educators or educational assistants, a curriculum, disciplinary knowledge, textbooks and teaching materials,

calculators, manipulatives, teaching strategies, assignments, exams, group work – in the situation or environment of that classroom. Personal practical knowledge is more than knowledge of theory; it is knowledge developed over time and arising from the teacher's experience which enables the teacher to make thoughtful, particular decisions about practice in a complex, dynamic and multifaceted situation.

Parents, in the same way as teachers, have personal knowledge gained from their vast array of life experiences. Parents, too, have practical knowledge gained from living in a contextualized situation – this time a home – with persons, things and processes – their children, other family members, possibly a pet, electronics, books, games, sports equipment – all interacting in dynamic and complex ways – the juggling of homework, lessons and leisure activities, meals, laundry, daily schedules and routines, a family outing, a birthday celebration. Parents' personal practical knowledge is knowledge gained over time and from their lived experience with their children, also in a complex, dynamic and multi-faceted context, which enables them to act and to make decisions in regard to their practices of parenting in informed ways.

When we return to the grade 8 teacher's statement, it becomes clear that this teacher believes parent engagement in mathematics is not possible because parents do not hold the personal practical knowledge of mathematics that teachers hold. What opportunities for parent engagement become possible, though, when our expectation is that parents come to their engagement in mathematics, not as teachers but as parents, with all the personal practical knowledge they have to offer as parents?

When my eldest son, Cohen, was in grade 12, he asked me to read a draft copy of an English paper he had written. I read his paper and, as I do with undergraduate and graduate students, I wrote in pencil in the margins – posing questions, asking for clarification, encouraging him to elaborate. When I handed his paper back to him and he began to read through it, his face fell. I asked him what was wrong. His words will stay with me forever. He looked up at me and said, "I don't want you to be my teacher. I just want you to be my mom." I approached his paper as a teacher, with the personal practical knowledge arising from my experience as a teacher. I attended to the development of his thesis statement over the unfolding of his paper, to the elaboration of his main ideas, to how he was supporting the claims he was making, and to the specificity of his language use. Instead, he wanted me to read his paper as his mom, with the personal practical knowledge of him that I have garnered over time. He wanted me to see his writing as a reflection of him, as someone with a growing voice and stance in the world; to be his greatest fan and applaud his efforts; to engage with him in a discussion of the kinds of ideas we so often entertain together. In that moment, Cohen wanted me to be his mom – and I am the only person who can be that for him. Parent engagement is not about expecting parents to do the work of a teacher. Parent engagement is about inviting parents to share and use their parent knowledge to enhance teaching and learning for their children.

Defining Parent Engagement

If parent engagement in mathematics is not about parents putting to use their knowledge of mathematics and

mathematics curriculum, what is parent engagement about? What might the use of parent knowledge to enhance teaching and learning look like in mathematics? First, let's take a close look at the word "engagement."

"Engagement" ... comes from *en*, meaning "make," and *gage*, meaning "pledge" – to make a pledge (Harper, 2002), to make a moral commitment (Sykes, 1976, p. 343). The word engagement is further defined as "contact by fitting together; ... the meshing of gears" (Engagement). The implication is that the person 'engaged' is an integral and essential part of a process, brought into the act because of care and commitment. By extension, engagement implies enabling parents to take their place alongside educators in the schooling of their children, fitting together their knowledge of children, teaching and learning, with teachers' knowledge. (Pushor & Ruitenberg, with co-researchers, 2005, pp. 12-13)

Now, let's examine the actual acts or practices which are often expected of parents by teachers and school administrators. During my years as a parent I have been asked to sign agenda books, initial marked assignments or exams, monitor or assist with homework, ensure my children have time and space to do homework, assist with or facilitate projects, attend parent meetings of various kinds.

Jeynes' (2005), [in a meta-analysis of the relation of parental involvement to urban elementary school student academic achievement] ... found it was not particular parent actions, such as attending school functions, establishing household rules, or checking student homework, which yielded the statistically significant effect sizes in the

research in relation to student academic achievement. Instead, it was things which created "an educationally oriented ambiance" (p. 262) – an attitude or an atmosphere which formed for the child a sense of standards or support – which produced the strongest results. (Pushor, 2007, p. 5)

My greatest contribution, then, as an engaged parent comes in the global ways in which I instill in my children the importance of learning, schooling and education, establish expectations for their performance in the broad range of schooling outcomes (e.g. attendance, behaviour, achievement, completion rates), and provide support and encouragement for their efforts. It does not come directly from the more specific actions I am typically asked to perform.

Being an Engaged Parent

In research on parent engagement and leadership I conducted with a team of co-researchers at Princess Alexandra Community School in Saskatoon, SK, I talked with one parent who described himself as an engaged parent. I was interested in learning from him what it was he was doing that he considered engagement. The first thing he said to me is, "I am working the fridge." When I told him I didn't know what that meant, he invited me to come to his home and see. In his home, he showed me a near-empty fridge, with only the kinds of basics – mustard and ketchup – that can be found as a staple in most fridges. When we sat at his kitchen table and talked, he told me a story of their family. He spoke about being on social assistance because of a physical injury he sustained many years earlier, and of the challenges of raising a family on the money he received. As we talked, I learned from him that "working the

fridge” meant finding ways to bring more food into the home without expending the little cash the family had. For this dad, it meant trips to the food bank for groceries or to the Friendship Inn to get bread. It meant working odd jobs to earn some kind of needed service in return, or to make some extra cash. The dad explained to me that the money he did not spend on food he used to put toward brand name jeans or shoes for his kids. He said:

My kids already have two strikes against them. They’re Aboriginal and they’re poor. Their chances of staying in school aren’t as good as other kids. If I can get them clothes that help them fit in, there’s a better chance they will go to school. So far, all my kids are still in school and they’re doing pretty good. (Field notes, March 21, 2005)

By working as hard as he did to ensure his children went to school and stayed in school, this father was creating for his children a strong sense of his support for their schooling and an expectation they would continue in school until graduation. Yves Bousquet’s thoughts on the improved student achievement results at Princess Alexandra Community School affirm this form of parent engagement. As a former principal of Princess, Yves spoke about how important student attendance and low levels of transiency are in a community school. “We can teach them if they are here” (Field notes, January 28, 2005). While this parent used his commitment and his personal practical knowledge to ensure his children were in school, the educators who worked with his children used their commitment and their personal practical knowledge to teach them well. This parent was an integral and essential part of the

processes of his children’s schooling; he was a parent who played a critical and complementary role alongside the educators who worked with his children.

Key Aspects of Parent Engagement

What this father’s story does is make visible some key aspects of parent engagement which are important for educators to consider. It is interesting to note that the parent defined his engagement in his own terms, not in relation to expectations for parent engagement established by a teacher or a school – attendance at meetings, signatures on school work, assistance or monitoring of homework. His engagement drew upon the personal practical knowledge he possessed as a parent – his life experiences, his knowledge of his community and its support systems, his knowledge of his teen children. It enabled him to do what he knew how to do, not to be positioned as a teacher in relation to his children. It did not cause him to feel unable or deficit, as often happens when a parent is asked by the teacher or school to do something, such as help with homework, that he may not know how to do or understand.

Another interesting notation is that this father’s engagement happened off the landscape of the school and yet it was integral to providing support to what was happening at school. So often, educators determine levels of parent engagement by counting the number (or calculating the percentage) of parents who attend a meeting, return a signed form, or participate in parent/teacher conferences. They count the parent actions they see, the things they know about – and these are, typically, the things they have requested of parents. What this parent’s story invites us to

consider is how much engagement in children's schooling is happening off the landscape of schools and how impactful that engagement may be. Jeynes' (2005) meta-analysis suggests that attending more closely to the kinds of things parents are doing to create an "educationally-oriented ambiance" in their homes and with their children may be an area worthy of investment if we want to promote and enhance parent engagement.

There is another thing that this parent's story foregrounds. In his story, the father says that he believes his children already have two strikes against them: they are Aboriginal and they are poor. His interest in having them stay in school is greater than course completion, achievement levels, or graduation rates, the kinds of things we often measure and report in schools and school systems. He wants his children to have a good life. He sees education as a means to employment and a way out of poverty. He sees education as a way for his children to break through stereotypes about First Nations individuals and to earn respected and rightful places in society. This father's engagement reflects an investment in his hopes and dreams for his children, and in their hopes and dreams for themselves. While teachers and administrators, in the roles they are in, are responsible to invest themselves in the processes and results of *schooling*, this father, in his role, is responsible to invest himself in the *education* of his children, something which includes schooling but is life-long and broader-reaching than schooling.

Let's return to questions I posed earlier in this paper. What knowledge, skills and attitudes does a mathematics teacher need in order to teach mathematics? What is the teacher's role

and what are the expectations of the teacher in regard to that role? It is commonly expected that the teacher knows mathematics and the mathematics curriculum, and has a philosophy and pedagogy to teach it. It is further expected that a teacher knows students – their prior knowledge, their current understandings and misunderstandings, and how to help them progress. I do not think it has been such a common expectation, albeit as important a one, that a teacher knows parents, families and communities, and the hopes and dreams they hold for their children. Because this has not been an expectation it has enabled teachers to assume, and to state, "Parent engagement in mathematics is just not possible."

What knowledge, skills and attitudes does a parent need in order to be engaged in a child's schooling in mathematics? What is the parent's role and what are the expectations of the parent in regard to that role? Perhaps the answer lays in the diverse, multiple and complex ways in which each parent creates "'an educationally oriented ambiance' (p. 262) – an attitude or an atmosphere which form[s] for the child a sense of standards or support" (as cited in Pushor, 2007, p. 5) for learning. Perhaps the answer lays in the ways in which the parent is able to bring his or her personal practical knowledge alongside that of the teacher to enhance both schooling and educational outcomes for his or her children.

Engagement Practices of Possibility

How might teachers learn about parents, families, and communities, and their hopes and dreams for their children in order to enhance parent engagement practices in relation to mathematics? How might teachers invite parents to

bring their personal practical knowledge together with that of the teacher's in order to enhance schooling and educational outcomes for students? This is complex work, especially in secondary schools where the schools are large and multifaceted, where students have multiple teachers, where teachers instruct large numbers of students, and where forms of communication, because of such size and numbers, tend to be general rather than personal. What I believe it will take is a rethinking and redesign of the structures and processes schools currently have in place, or a substitution of old practices for new, more effective ones.

“Ask Them” – About Themselves

Often educators assume they know the families with whom they work and the communities in which they work. Their knowledge generally comes from their experiences in a school over time, from administration and/or colleagues, or from school or neighbourhood profile information. Rarely do educators ask parents to tell them about themselves, their families or their communities. “Who are you? Where do you come from? What are your goals, aspirations, dreams for your children? What do your families/your children do at home/outside of school?”

When educators have a comprehensive picture of the lives of their students, and the students' family and community members, they can better understand the supports students have for learning – and those they may not have. Knowing about such things as the books and resources families have in their homes, the educationally-enriching activities they engage in, the places they visit (Reynolds & Gill, 1994), and the forms and varieties of literacies they make use of provides an important

backdrop for curriculum development and delivery. This knowledge can inform changes to everything educators do in schools from the structuring of homework policies to lesson and unit planning, from parent communication to the analysis of student achievement result data.

This kind of learning about families does not have to be the work of each individual mathematics teacher or even the work of a mathematics department; it can be done as a school-wide initiative with value for all teachers and staff in the school.

Registration forms. Each new school year, I complete a registration form for my children on which I am asked to record or update our family demographic, contact, and medical information. Using this same form and process, but with a cover letter explaining in detail the purpose of the request, the school could ask contextual (and optional) questions, such as those above, that invite rich information about families. This information could then be shared with all teachers and staff who are connected to a student/family.

Staff meetings. As a regular agenda item at each staff meeting, parents from a representative group in the school community could be invited to present for a few minutes and converse with staff about an aspect of their lives and families: their culture, language, religion, experiences as refugees or immigrants to Canada, the community's history, shifts they have observed in their neighbourhood, community services and amenities which are available, challenges their neighbourhood is facing, and so on.

Community walk/drive. In lieu of a staff meeting or as a professional development session, designated parents and/or community members could lead staff on an excursion through the neighbourhood to highlight amenities, services, shifting demographics, points of note, strengths and challenges within the community. The intention would be to invite educators to see the community through the eyes of individuals who reside within that community and to hear their stories of where they live.

“Ask Them” – How They Want or Are Able To Be Engaged

Given the multiple family structures that exist today, and the wide-ranging diversity that exists within them, it is apparent that families create “an educationally oriented ambiance” in their homes which aligns with their aspirations for their children, their personal practical parent knowledge, and the context in which they live. Hill, Tyson and Bromell (2009) see the creation of this ambiance, or what they call parents’ “academic socialization” (p. 58) of their children, being generated through the various ways in which they “communicat[e] expectations for and the value and utility of education, link[] schoolwork to current events, foster[] educational and occupational aspirations, discuss[] learning styles and strategies, and mak[e] preparations and plans for the future” (p. 58). For the father whose children attended Princess Alexandra Community School, this meant “working the fridge.” For me, it has meant reading together with my boys, playing card and board games, and taking family trips. For my friend Lorayne, it has meant placing a strong emphasis on her children’s participation in organized sports activities. For my friend Jan, it has been participation with her daughter

in Chinese school and cultural events. For my friend Bonnie, it has meant time spent with her family in nature and the outdoors. In their research, Reynolds and Gill (1994) found that the provision of educationally-enriching activities, materials and experiences at home enhanced students’ achievement while, in contrast, “there is no good evidence that homework produces better academic achievement” (Kohn, 2006, p. 27). In fact, Hill, Tyson and Bromell (2009) found “homework help may undermine achievement when it interferes with students’ autonomy, results in excessive parental pressure, or is not consistent with how materials are presented at school” (p. 58). Returning to the importance of engaging parents in ways that draw on their personal practical knowledge *as parents* and that enable them to shape how they are engaged in their children’s learning and growth, it becomes important to ask them how they may want to use their parent knowledge alongside teacher knowledge as they work to support and nurture their children’s learning.

Changed Practices in Schools That ‘Asked Them’

At H.J. Alexander School in Toronto “almost half of students do not speak English as their first language, more than half come from low-income households and just 20% of their parents went to university” (MacDonald, 2009). In getting to know the parents of their students, staff at H.J. Alexander heard stories of the challenges many families faced to get to Canada and of the hardships they were enduring to provide better educational opportunities and a better life for their children. Being told by parents of how high their hopes and expectations were for their children’s achievement, as a critical means to the

better life they sought for them, staff put a number of structures in place. These ongoing structures include curriculum nights, organized by grade level, in which teachers explain to parents “what their children are working on and what’s needed at home to help them” (MacDonald, 2009). They share samples of achievement tests two months prior to the evaluation period so parents know what the test may be like and what different levels of achievement on that test look like. In conjunction with the grade level sessions, parents are also offered opportunities “to attend meetings to brainstorm customized teaching strategies to help their own child.” In speaking with parents when I visited H.J. Alexander School in November 2009, they expressed how pleased they are with the school’s recognition of their parent voice and the inclusive ways in which they partner with them in the core teaching and learning work of the school – the establishment of goals and learning targets, the discussion and teaching of strategies for curriculum implementation, the analysis and interpretation of achievement results. In 2003, 29% of H.J. Alexander School’s grade 3 students met the standard in math. In 2009, with the support of an engaged community, 82% of the children met the standard (MacDonald, 2009). Parents and staff have determined ways to engage together that suit them and their particular context and are producing the results they desired.

At Military Trail School in Scarborough, Ontario, over 60% of the students speak a primary language other than English, and approximately 20% of the children have lived in Canada five or fewer years (Toronto District School Board). Many families live in a dense area of high rise apartments near the

school. Because of parents’ employment in the evenings in shift work or second jobs and/or because of concern for their children’s safety, many families require their children to remain in their apartments from the time they arrive home from school until they leave for school the next day. In response to their knowledge of the community and in relationship with parents, school personnel organized homework clubs on site immediately after school in which students are able to do their homework with assistance. Much of the tutoring is done by teachers in the school, individuals who know the children’s curricular and learning needs well. (Teachers who choose to provide this tutoring are paid for their services.) As was apparent in my visit last November, parents in this community care as deeply about their children and their children’s success as parents do in the H.J. Alexander community. Because their circumstances make it hard for many of them to support their children with their homework, together staff and parents have determined an alternative, yet reciprocally beneficial resolution.

Using Knowledge of Parents and Families to Change Your Practices

Open house sessions in schools are common and are typically used by teachers as an opportunity to provide parents with information about the curriculum, assignments and expectations, schedules and procedures. Adopting an “ask them” stance, such sessions can be used, instead, for meaningful communication and knowledge sharing. In designing such an interchange, teachers create an occasion in which to make the mathematics curriculum accessible to parents and to share their approach to teaching mathematics. Parents have an opening,

armed with more and better information, to discuss ways they currently support their children and to generate new possibilities for their engagement in their children's learning. Focal questions which may be posed to frame such knowledge-sharing sessions include:

- What decisions can we make together, as teachers and parents, using our combined knowledge?
- What do we, as parents, want to learn from you as teachers?
- What do we, as teachers, want to learn from you as parents?
- What do we, as parents, want to learn from other parents?
- What do we, as parents and/or as teachers, want to learn from students?

Such discussion creates space for and honours the important contribution all stakeholders make in the learning process.

From the responses to the framing questions, teachers and parents can work together to plan and schedule a series of relevant sessions to be held over the course of the school semester or year. Teacher and/or parent facilitators and/or student facilitators can volunteer or be sought out to lead the sessions, and times and locations for the sessions can be mutually-determined. Of the scheduled offerings, some may be topics of general appeal (e.g. understanding the mathematics curriculum) while others may be topics that address specific areas of interest or relevance to only some parents (e.g. second language learning in mathematics).

From community to community, school to school, how parents and teachers respond to the framing questions will differ, depending on who

they are, on what their context is, and on how they understand their roles in relation to the teaching and learning of mathematics. Here, I am providing some imagined possibilities for interactive sessions only to make more concrete this rethinking of parent sessions which I am proposing. I invite you to accept them as prompts for your own particular imaginings.

Shared Decision-Making

- Talk with parents about homework and mutually determine a homework policy. What is their experience with helping or supporting their children with homework? What has worked/not worked for them in the past? What kinds of homework would they like to see their children doing? How much? How often? Share your beliefs and expectations about mathematics homework. Find a place of common agreement. Just as with instruction, differentiation is possible here. There may be a parent or two who, for whatever reason, find they cannot commit to an agreed upon policy. Negotiate those agreements individually, acknowledging the differences which exist between students, parents and families.
- Join with parents to analyze student achievement data. Frame the conversation around the shared responsibility of parents and teachers for and the shared contribution to student achievement. Together ask the questions, "Where are our children doing well? Where do they need more support? What are the implications of this information? What do we do now?"

Parents Learning from Teachers

- Model a mathematics lesson for parents (or show them a video of a

lesson you taught to their children). Knowing the steps of a lesson and the teaching processes you use, parents are more able to talk meaningfully with their children about their mathematics class or to provide guidance and encouragement for them as they work with concepts at home. While the parents' role is not to teach the math, or even know the math, being more knowledgeable about how to help their children strategize, persevere, or figure out what to do when they do not know what to do enhances parents' engagement in their children's learning.

- The constructivist approach to mathematics is new to many parents who were taught mathematics in procedural and operational ways. An opportunity to observe you work with a mathematics strategy they have not experienced themselves – sharing solution strategies, using manipulatives, recording mathematical thinking in logs or journals – enables them to become a part of their children's curricular learning. Observing a lesson or watching a student demonstration, with the opportunity to interact with the students while they work, orients them to their children's learning experiences. It shifts them from being 'outsiders' to the mathematics classroom to being 'insiders.'

Teachers Learning from Parents

- Invite parents to share their own school experiences with math, their sense of comfort with this curriculum area, the ways in which they engage with their children mathematically, whether through school work or outside of it. This insight will enable

you to come alongside parents and students in ways that are responsive and particular.

- Invite parents to share their knowledge of their children as learners of mathematics – their attitudes and dispositions at home towards mathematics, their approach to homework, the satisfaction or frustration they display, the nature of their conversation about their math class or content, the kind of assistance or support they seek. Given the more intimate nature of a home, students often express their thoughts and feelings differently with family than they do in the more structured and formal setting of a classroom. As a result, parents often have insights and perceptions of their child as a learner that can complement those of the teacher.

Parents Learning from Parents

- Invite parents to organize a family mathematics games/puzzles night at which families can share, teach and exchange with each other games/puzzles they play at home that have a mathematical aspect to them.
- Invite parents to host informal gatherings through which they create a network of support and information for one another. The gatherings can provide an opportunity for parents to learn from each other's lived experiences and parent knowledge. From sharing experiences with their children's transitions to high school or postsecondary education, to understanding courses or program offerings, to letting others know about extracurricular or supplementary clubs or activities, to discussing ways of responding to special learning or language needs,

the topics parents may choose to discuss are perhaps endless.

Parents Learning from Students.

- Organize student sessions in which parents observe students sharing and teaching each other learning supports such as study strategies, organizational skills, review processes, or problem-solving approaches to mathematical challenges.
- Book some time in a computer lab where students can explore with their parents valuable websites which provide information on such things as mathematical content, strategies, tutorials, solutions demonstrations or examples, as resources which may be useful to them at home.

Parent Engagement in Mathematics Is Possible

Underlying all of these imagined possibilities for new practices is a redefinition of what parent engagement is, who or what the engagement is with, and what forms it may take. Critical to believing that “parent engagement in mathematics IS possible” is an understanding that the very nature of being a parent means that a parent will play a role in a child’s learning different than that of a teacher. It is an understanding that a parent will bring to the teacher-student-parent-curriculum relationship valuable parent knowledge – particular knowledge about children, family, and context and particular knowledge about teaching and learning – but will not necessarily bring knowledge about mathematics curriculum or concepts. It is an understanding that a parent’s most significant contribution falls outside of the types of engagement which educators have typically requested or expected of parents, and instead is

comprised of the kinds of things a parent does at home and in life to establish for a child a sense of support and expectations for learning. Critical to believing that “parent engagement in mathematics IS possible” is an understanding that what parents are actually engaged with is their children, their children’s teachers, their children’s learning in relation to mathematics – not with the mathematics itself.

Underlying all of these imagined possibilities for new practices is also a redefinition of the role of a teacher. Due to increasingly complex social factors, schools and teachers are not able to address solely through traditional ways the challenges they face to enhance students’ academic achievement or their attainment of other educational outcomes (Wotherspoon & Schissel, 2001). Elements which shape the contexts of families and communities – such things as housing and food security, adult literacy, education and employment, culture and language, health and safety – impact student learning, wellbeing, identity, attendance and retention. Schooling necessarily then requires the contributions of parents, families, and community members, alongside those of educators. Correspondingly, working with parents becomes an essential aspect of a teacher’s role.

There is a great deal of research in this field, done over a number of years and by a broad range of scholars, which shows that engaging parents, families and communities in their children’s learning contributes to student’s academic and social success. In a comprehensive review of this research, Henderson and Mapp (2002) conclude:

The evidence is consistent, positive and convincing: families have a major influence on their children's achievement. When schools, families, and community groups work together to support learning, children tend to do better in school, stay in school longer, and like school more. (p. 7)

Hill, Tyson and Bromell (2009), who specifically research parent involvement during middle school, confirm that "parental involvement still matters for adolescents' achievement" (p. 58). Parent engagement correlates with such performance traits as better grades, enrolment in higher level programs, higher graduation rates, a greater likelihood of enrolment in postsecondary education, and the development of realistic plans for the future (Henderson, Mapp, Johnson & Davies, 2007).

To take up this aspect of their work, to invest themselves in engaging parents and families, necessitates a commitment on the part of educators. It is a commitment to working in relationship, to developing shared hopes and aspirations, to acting on the collective knowing of students, parents and teachers. It is a commitment that recognizes that children benefit in academic and social ways from their engagement in the informal teaching and learning of the home just as they do from the formal teaching and learning in school. It is a commitment that recognizes how important it is for educators to know about and capitalize on the learning opportunities that extend off the school landscape and beyond the hours of the school day. It is a commitment worthy of the rethinking and shifts in practice it entails. And, it is a commitment that enables us to conclude, "Yes, parent engagement in mathematics IS possible."

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THE PROFESSIONAL DEVELOPMENT DILEMMA IN MATHEMATICS

Yvette d'Entremont and H  l  ne Gendron

As technology advances at an exponential rate, access to information is, literally at one's fingertips. The iPhone 3GS is a perfect example with its capabilities of video, camera, voice control, compass, landscape keyboard, messages, internet, etc. The phone even has a feature of "cut, copy, paste". Who needs a computer! Are teachers able to keep up with students who have immediate access to information?

Professional development is one way to help teachers keep abreast of changes, within the curriculum and outside the curriculum, and to provide ways to cope with the ongoing changes. Guskey (2002) indicates that the intention of professional development is to bring about change in the classroom practices of teachers, in their attitudes and beliefs, and in the learning outcomes of students. In Alberta, there are ample professional development opportunities. The Alberta Teachers' Association (ATA) assures annual professional development conferences, Alberta Education recognizes its responsibilities regarding professional development, the six regional consortiums and (since 2007) the provincial francophone consortium all play a major role in providing resources and support related to the implementation of the new mathematics programs. Schools also organize professional development days and Alberta teachers must also prepare an individual professional development plan. To complete this smorgasbord of professional development opportunities, publishing houses also offer professional development workshops.

Despite these numerous opportunities, certain teachers consider professional development days a waste of time. Dionne (1988) identifies four problems that affect mathematical pedagogy: the insistence on symbolism, the strong influence of formalism, the invading presence of learning theories, and the large number of mathematics teachers for whom teaching means getting an answer. How does professional development respond to these issues?

The most common type of professional development is the workshop. Garet, Porter, Desimone, Birman and Yoon (2001) indicate that "although traditional forms of professional development are quite common, they are widely criticized as being ineffective..." (p. 920). This criticism stems from the fact that traditional forms of professional development, like the workshop, fail to provide teachers with "sufficient time, activities, and content necessary for increasing teachers' knowledge and fostering meaningful changes in their classroom practice" (p. 920). Kajander and Mason (2007) also indicate that traditional professional development is ineffective and "the ineffectiveness of traditional professional development is well characterized in the literature..." (p. 417). In another study, Borko (2004) concludes that, "despite recognition of its importance, the professional development currently available to teachers is woefully inadequate" (p. 3).

The structured workshop approach to professional development that occurs outside the classroom may not be the ideal approach to professional development as these experiences should have a "substantial positive influence on teachers' classroom practice and student

achievement” (Garet et al., 2001, p. 917). Although teachers are encouraged to change or improve their mathematics teaching, “their personal beliefs, knowledge, confidence and personal intentions towards growth and change are all complex factors which may influence [their] capacity, and their decisions about personal change in their teaching” (Kajander & Mason, 2007, p. 417). Curriculum reforms, and new programs of study, ask teachers to make complex changes to their teaching and “such changes require that teachers reinvent their practices” (Franke, Carpenter, Levi, & Fennema, 2001, p. 654). This is not an easy task and presents a challenge to most teachers. As such, and in order to help teachers engage in professional development activities, we must encourage them by providing activities that “supports their ongoing learning and simultaneously provides opportunities for (them) to create collaborations with their colleagues” (Franke et al., 2001, p.686).

Experience with professional development activities in mathematics has led us to believe that there are more professional development activities in mathematics than in any other subject. This observation is confirmed by Borko (2004), Riley (1994), Gates and Vistro-yu (2003), and Crayen and Lahn (2009) who indicate that the number of professional development activities are based on the perception that mathematics, science and literacy are the academic pillars of success. Borko (2004) also supports our belief that there are fewer professional development activities at the senior high school level than at any other level.

Normally, professional development activities are carried out by one’s peers. Borko (2004) refers to this as *teachers*

teaching teachers. However, there seems to be fewer and fewer peers available and willing to offer professional development programs to colleagues at the senior high school level. We have seen an increase in the number of colleagues offering professional development at the elementary level, but the opposite trend is true at the senior high level. In general, teachers are comfortable leading professional activities for colleagues who teach at the same grade level or lower. Our experience with professional development programs is that teachers are uncomfortable leading workshops or other professional development activities to colleagues teaching at higher levels. This might be the cause of fewer professional development activities available to senior high math teachers.

Perhaps this situation can be remedied by facilitating discourse between senior high math teachers and post-secondary math professors. However, this might be difficult to achieve due to the fact that high school math teachers must cater to a specific program of studies whereas the post-secondary professor does not. Thankfully, postsecondary mathematics educators volunteer often to provide support.

Another disadvantage is that the high school teachers do not have easy access to postsecondary professors. Frequently, they do not even have access to other senior high math teachers. Large city high schools usually have more than one senior high math teacher but such is not the case for small city schools and rural schools. In many cases the high school math teacher stands alone in his/her school. This is especially the case in francophone and immersion schools. The geographical location of

francophone schools in Alberta provides another obstacle as francophone schools may be hundreds of kilometers apart. This geographical dilemma may be solved using technology. Technology can play a major role in professional development activities for colleagues that are at a distance, allowing for colleagues to meet virtually and dialogue.

Supported by technology, the face of professional development is changing. Last January, the seven Alberta consortia gathered over 600 teachers linked by 12 video-conferencing sites and via Elluminate (for the Francophone teachers spread out across the province) for a day of professional development. This was the beginning of several activities to ensure a continuation of professional development for mathematics teachers. Virtual meetings, MOODLE sites, and other follow-up activities changed a one day activity into a continuation of interconnected professional development opportunities.

Kajander and Mason (2007) suggest the *Professional Learning Group* model as a means to successful professional development. The Professional Learning Group is composed of colleagues meeting on a regular basis and “setting their goals collaboratively and continuously negotiating how they will address those goals” (p. 419). Professional Learning Group discussions can focus on:

integrated curricula, where the three components of knowledge base [knowledge of mathematics content, knowledge of student epistemology, and knowledge of pedagogy] are addressed in a synergetic manner, [and] can help teachers grapple with the mathematics and at the same time

reflect on their own learning, which, in turn, can help them appreciate epistemological and pedagogical issues. (Harel & Lim, 2004, p. 32)

Research indicates that “sustained and intensive professional development is more likely to have an impact...than is shorter professional development” (Garet et al., 2001, p.924). Professional development activities, such as the Professional Learning Group, must be sustained over a period of time so that the change may translate into more effective student learning opportunities. This model could be beneficial to senior high math teachers.

The big question is how to make it all happen in the reality of the different types of support necessary for different schools comprised of teachers who possess distinct competencies and different learning theories. This dilemma is reflected in the conclusion by Garet et al. (2001), “if we are serious about using professional development as a mechanism to improve teaching, we need to invest in activities that have the characteristics that ... foster improvements in teaching” (p. 937). He also indicates that “activities that extend over time are more likely to allow teachers to try out new practices in the classroom and obtain feedback on their teaching” (p. 921). After all, “the continual deepening of knowledge and skills is an integral part of the profession” (p. 916) and all stakeholders have a role to play in the professional development of teachers. Could technology be the key to more effective professional development? The use of technology could allow educators to participate in professional development activities no matter where they are.

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THE ‘ETHANOL’ ACTIVITY

Tyler Pokoyway, Stacy Renneberg and Rick Seaman

With the push for renewable resources and cleaner energy sources, two secondary mathematics education interns in the fall of 2007 developed an activity investigating the use of ethanol as an energy source. After the first intern had taught the lesson at their school, both interns met to discuss how the lesson could be improved. The ‘new and improved’ lesson was then taught at the second intern’s school. This article will demonstrate how mathematics an “inch deep” based on, in this case, proportional reasoning can become a cross subject area focus on real-world issues which are a “mile deep”.

Background

The following is a cross-discipline learning activity focused on the production of ethanol for fuel in the province of Saskatchewan, Canada. With the environment being a ‘hot’ topic all over the world, there is a push for renewable resources and cleaner energy sources. One such energy source is ethanol and because some of the ingredients used in making ethanol are corn, grain sorghum, wheat, and barley there has been a push to use Saskatchewan’s agricultural resources for its production (Enterprise Saskatchewan, 2009).

During the fall semester of 2007, two secondary mathematics education interns, in the Faculty of Education at the University of Regina, developed a mathematics lesson concerning the use of ethanol blended gasoline that involved other subject areas such as english, science, and social studies. Both interned at the high school level at different schools in Regina, the capital

of Saskatchewan. The idea was that after the first intern had taught the lesson they would meet and discuss how the lesson went so as to make improvements to it. The intern who had not taught the lesson yet would then take the ‘new and improved’ lesson and teach it to their class. What follows describes the mathematics lesson developed called the ‘Ethanol’ activity and the collaborative efforts of the two interns.

‘Ethanol’ Activity

In 2007, Saskatchewan gas pumps were required to blend 1% ethanol into their total gasoline sales and today the figure has been increased to 7.5% (Enterprise Saskatchewan, 2009). The government claims that this renewable fuel will not only improve the economy but also protect the environment (Saskatchewan Ministry of Enterprise and Innovation, 2007). However, there are many critics against the use of ethanol as a way to protect our environment and claim that ethanol causes more problems than benefits. The debate has begun and there are many players on both sides of the issue in both Canada (Coxworth, 2006) and the United States (HybridCars.com, 2007). (Other sources of energy such as wind power, solar, and geothermal energy are also encouraged in Saskatchewan.)

First School

The lesson began with a mock scenario that had students acting as candidates running for the position of Premier of Saskatchewan, with voters wanting to know where they stood on mandating the use of ethanol-blended fuel. The task was for them to sit down with their advisors (their group) and go through the numbers and read the research on the issue. Then they were to make an informed decision on whether

or not they supported the move to use provincial agricultural resources for the production of ethanol. This scenario served as a hook to engage students into the 'Ethanol' activity. The objective was to introduce the social, environmental, and ethical issues surrounding the ethanol debate during english class (see Figure 1.). Then students in their mathematics class were to apply proportional reasoning to interpret related data and then make a list of pros and cons to help them substantiate their decision of whether or not they supported the production of ethanol for fuel.

1. If ethanol is made from wheat and corn, what will that do to the selling price of wheat and corn crops? Will this affect you and your family at the grocery store? What if your family was growing these types of crops?
2. How could the province provide the cropland needed for ethanol production while still producing the amount of wheat needed for human consumption (food for people) and feedstock (food for farm animals)? Is there enough land area for the ethanol crops?
3. Are there any international issues/effects as a result of producing crops specifically for ethanol production? Explain.
4. If our demand for/consumption of oil/gasoline is increasing and considering the amount of CO₂ from the producing ethanol, does using ethanol as a fuel source actually reduce CO₂ emissions? Explain.
5. Instead of producing more environmentally friendly fuel, what other environmentally friendly

alternatives could we do to meet our transportation/energy needs?

Figure 1. Issues/Dilemmas

English Class

English class began with finding out what the students already knew about ethanol and then explaining the 'Ethanol' activity to the students. The english component consisted of a resource package of Issues/Dilemmas discussion questions (see Figure 1) and five articles from the local newspaper (found at the following website: <http://www.canada.com/reginaleaderpost/index.html>). One article, written by the financial editor Bruce Johnstone (2006), discusses the pros and cons of the renewable fuels industry and whether tax dollars should be used to subsidize the industry. The second article, written for the Associated Press (2007), describes the impact a corn-based ethanol boom in prices would have on the corn and related industries in the United States. A third article, written by an agrologist consultant and Saskatchewan farmer Kevin Hursh (2007), discusses the impact the renewable fuels industry will have not only on the environment but the grain industry and rural development. The last two articles are letters to the editor; one in response to production of ethanol for fuel (Kurtenbach, 2007) and the other (Leader, 2007) questioning what drivers are willing to do to reduce greenhouse emissions and what is spent on gas.

Students were placed in groups of five, with each group member taking a different article in the resource package to read and point out what they thought were the main ideas in the article. Then each group member discussed the article they read within their group so that everyone could understand the issues

contained in the articles. Then the group responded to the Issues/Dilemmas questions (see Figure 1). A class discussion about these questions followed ending with a brainstorming session about the possible alternatives to using automobiles for transportation and burning fuel for energy. The english component took place over two, one-hour class periods.

Math Class

The students continued the 'Ethanol' activity in their mathematics class, in the same groups, with the previous resource package supplemented with: two more articles from the local newspaper that discussed the construction of ethanol production plants in Saskatchewan and supply side concerns (MacAfee, 2006; Johnstone, 2007); a page which compared the benefits and environmental impacts of ethanol fuel, in Brazil and in the United States (Dias de Oliveria, Vaughan, & Rykiel Jr., 2005); a map of Saskatchewan Crop Districts and Rural Municipalities (Saskatchewan Agriculture, 2006); information on how ethanol is made and where it comes from (Integrated Grain Processors Co-operative Inc., 2005) and agricultural data from Statistics Canada (2006 to 2008). The students used their supplemented resource package to find the facts necessary to answer the questions on the DATA sheet (see Figure 2). After every group had completed their investigations, they discussed the implications of their findings and how they tied into the issues discussed previously during english class. The groups then created a list of pros and cons to help them support their position. Each group member was to come up with their own decision as to whether or not they supported the province's decision to

produce and blend ethanol into gasoline. As an exit slip each student was to answer the questions: "What do you know now about ethanol and the ethanol debate in Saskatchewan that you didn't know before? As an informed citizen what can you do about this issue to express any concerns or issues you have?" Interestingly one of the students responded with: "I learned that ethanol might not be good for our environment because it takes a lot of crop land and it makes carbon dioxide."

1. Find the ratio of the amount of CO₂ emissions from the production and consumption of ethanol (in tonnes) to the amount of ethanol produced (in litres), if the average amount of CO₂ emissions is 1.43 tonnes for every 1000 litres of ethanol produced.
2. How many cars could drive for a year and emit the same amount of CO₂ emissions as that which would be emitted by producing the required amount of ethanol that Husky Energy and Terra Grain Fuels claim they will produce in a year? Note: The average passenger car produces ≈ 5.20 tonnes of CO₂/year (US Environmental Protection Agency: Office of Transportation and Air Quality, 2007).
3. If Saskatchewan produced 18.6 million tonnes of wheat last year, find the ratio of the amount of wheat needed for ethanol (in tonnes) for one year to the amount of total wheat produced (in tonnes) in 2006. Note: 1 bushel $\approx .027$ tonnes.
4. What is the percentage of wheat needed for ethanol of the total amount of wheat produced last year in Saskatchewan?

5. How much crop area would be needed to produce the wheat/corn for ethanol production?

Figure 2. Data

Second School

Improvements were made to the 'Ethanol' activity at the first school and adaptations were made considering the differences in student abilities in the second school. Improvements and adaptations made to the lesson included: adding a science portion to the activity; the math lesson would take place before the english lesson in order to accommodate the debate portion at the end of the activity; inclusion of additional resources (e.g., YouTube video, smart board demonstration); a guest speaker from the provincial government; and a pro-ethanol and anti-ethanol class debate.

There were also instructional adjustments needed to accommodate the learning needs of each group of students as the lesson was transferred from the first school. A revised question worksheet was included to help the students answer the mathematics related questions. For a sample question see Figure 3.



Figure 3. Sample: Revised question worksheet.

Science Class

A large Nimbus water bottle and a flame were used to demonstrate the complete combustion property of ethanol to the science 10 class (Metacafe Inc., 2007; Saskatchewan Learning, 2005). The demonstration served as an introduction to ethanol and its

combustion process. Since the students knew very little about ethanol and its applications it turned out to be a great way to introduce the topic. One student stated "I learned that ethanol is an alcohol that we use for fuel." The students became absorbed in the topic and asked questions about other options for reducing pollution and why our government would mandate ethanol if there were other 'greener' options to research. Everything from hydrogen fuel cells to wind power was discussed with one student suggesting that "we recycle older vehicles and replace them with hybrid vehicles."

Math Class

This time the math lesson took place before the english lesson in order to enhance the debate portion at the end of the activity. Both teachers noticed that the students from the first school were anxious about the proportional reasoning associated with the assignment and that they had some trouble with the wording of the questions on the questionnaire (see Figure 2). Consequently, a guided answer sheet was created with helpful hints to help students solve the problems (see sample Figure 3). To guide the students through the math questions, the first question was read together and the teacher showed them how to use the resources provided. It was hoped that by working out the questions with the students it would reduce their math anxiety and help them stay focused on the activity and the debate preparation. The map that showed the crop districts and rural municipalities in 2004 (Saskatchewan Agriculture, 2006) was used with smart board technology to suggest what might happen if there were areas of drought, flooding, or fire and how this could affect the land area available for farming. This helped the

students see how using crops for fuel might affect our available food resources.

English Class

Following math class the students discussed the Issues/Dilemmas questions (see Figure 1) in English class. This class began with the YouTube video (halcollins0, 2006) that illustrated the process of ethanol production and how some race-cars now use 100% ethanol fuel. This turned out to be a great starting point for the discussion. For example, one student asked, “Why can’t all cars run on 100% ethanol?” Added another, “We should all use ethanol, or stop driving so much.” The format of the english lesson was the same at both schools except this time the civil servant who wrote the ethanol mandate for Saskatchewan was the guest speaker. He spoke on the benefits and faults of using ethanol as a fuel source, which added a dynamic feeling to the classroom. The students became comfortable enough to ask him questions like, “How much land would we need to run our vehicles on 100% ethanol?” They asked him his opinion concerning the questions on the Issues/Dilemmas sheet (see Figure 1).

The students were then separated into pro-ethanol and anti-ethanol groups, based on their stance on the issue. Because the students were unfamiliar with how to run a debate, a package was created called *How to Run a Debate Information* that included a rubric created by one of the interns (see Figure 4), debate guidelines (LifeBytes, 2003), and a debate guide which asked for the following information: the student’s position; main points for your argument plus the source; main points against your argument plus the source; interesting facts to consider when time to vote; things I haven’t thought about; do I

agree with the position I took? Why or why not?; would I like to investigate further?

Name:

Group Viewpoint:

Comment:

Final Score:

	100%	80%	60%
View point	Viewpoints are clear and organized	Most viewpoints are clear	Viewpoints are unclear and disorganized
Use of facts and examples	Arguments are supported with facts and examples	Most arguments are supported with facts and examples	Arguments lack factual support
Relevance of supporting arguments	All supporting arguments are relevant	Many, but not all, supporting arguments are relevant	Few supporting arguments are relevant
Strength of arguments	All arguments are strong and convincing	Some arguments are convincing	Arguments are not convincing
Speaking voice	Voice can always be heard	Voice is heard most of the time	Voice is difficult to hear
Preparation	Student is well prepared	Student needs more preparation	Student is unprepared to defend argument

Figure 4. Debate Rubric

To further facilitate the debate, the guest speaker was invited to help out each side of the argument with the two interns helping the students think about different arguments the other side might present in order to prepare a rebuttal. The students took 15 minutes to go over their pros and cons, and to write their opening comments.

The debate lasted 30 minutes. Each student had a role to play, some were the speakers, and others gathered information or pointed out something from an article that could help their side

of the argument. One student had the role of asking the other side to “PROVE IT”.

After the debate the students had an idea of where they stood and worked independently to finish the questions on Figure 1. After the debate one student concluded “I love arguing, that was fun.”

Additional Student Comments

The same exit slip was handed out again asking students to respond to the questions: “What do you know now about ethanol and the ethanol debate in Saskatchewan that you didn’t know before? As an informed citizen what can you do about this issue to express any concerns or issues you have?” Some typical responses:

- “Ethanol pollutes just as much as regular gasoline, when it comes to carbon, we should try other energy methods.”
- “It is good that we can use ethanol for fuel, because we are using all of our fossil fuels up”
- “We will need a lot of farm land to make ethanol, so I don’t know if Saskatchewan can support ethanol as a fuel. Ethanol is good because plants absorb carbon emissions, but we need to drive tractors to get the crops for ethanol anyway, so I don’t know if it is a good or bad idea.”
- “We can just invade [the province of] Manitoba if we need more land!”
- “Why don’t we just make hybrid cars mandatory? Then we wouldn’t need as much fuel for transportation.”

Reflections

The students were not well informed on the ‘ethanol debate’ prior to this activity. As the ethanol topic was

explored, and the students began reading the articles and discussing the issues, they became involved and asked many questions. Many of the students not only wanted to know about the use of ethanol, but asked about other options for renewable fuels. As a result of this activity an excellent discussion ensued regarding renewable energy and other alternatives for transportation and energy sources.

At the first school, it was thought that the time lapse between english and math lessons was a little long. It would have been more beneficial if the lessons had occurred within the same week or the next day as happened at the second school. Also when the ‘Ethanol’ activity was revamped at the second school for the science and math classes, the students were able to stay focused on the activity during two back-to-back periods.

The science curriculum was a good fit for this activity because the lessons involving hydrocarbons, finding formulas, making chemical equations, and the properties of different substances lent itself to using ethanol. As the debate and discussions developed it became apparent that the ‘Ethanol’ activity could easily involve not only mathematics, science and english classes but also social studies, and physical education classes.

Looking back we found that bringing in a government official to address the provincial ethanol mandate into the english class at the second school was beneficial. After reading the articles and having a class discussion the students were, we argue, well prepared to ask more relevant questions of the speaker. Bringing the speaker into the debate also brought another opinion and new

information for the students to consider. As interns we also benefited from the presentation probably as much as the students did, as we learned that the use of ethanol-blended fuel was mandated in Saskatchewan for economic reasons and not just environmental reasons. The debate also added an exciting dynamic to the lesson, as a student wondered if ethanol was the best option (see Figure 5).

I personally am unsure as to support the use of ethanol. If I was to decided now, I would say...that we don't use our corn or wheat because we need that for other purposes. ~~and~~ People could take public transportation or car pool more often

Figure 5. Sample student response

The 'Ethanol' activity demonstrates how mathematical content an "inch deep" involving, in this case, proportional reasoning can be utilized to connect a variety of subject areas while making learning not only relevant but a "mile deep."

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THE SLOPE OF THE ANGLE BISECTOR RELATIONSHIP IN APPLIED AND THEORETICAL PROBLEMS

Gregory V. Akulov

The basic math terms of “slope” and “angle bisector” are included de Klerk’s (2009) elementary school *Math Dictionary*. A derivative topic, the “slope of the angle bisector,” might be expected to appear in a high school math. A century ago, to learn how to find the slope of the angle bisector, students had to take a university math class (Kindle, 1950). Even thirty years ago, I learned this topic in my first year at university. However, today the concept of the “slope of the angle bisector” is ready to be introduced in high school math because of a concise and suitable relationship that has been found.

If m_1 and m_2 are the slopes of two intersecting lines, and m_{bis} is the slope of the line that bisects angle between them (see Figure A), then:

$$m_{bis} = \frac{a}{b \pm \sqrt{a^2 + b^2}}, \text{ where}$$

$$a = m_1 + m_2, \quad b = 1 - m_1 m_2. \quad (1)$$

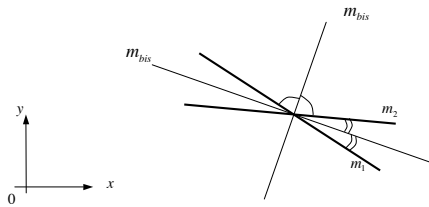


Figure A.

How nicely algebra and geometry are integrated in this relationship! Note that it was found and first published in Canada (Akulov, 2009).

Various problems in science, engineering, and mathematics require

the slope of the angle bisector to be calculated (Cummings & Laws, 2004). Let’s consider some applications (while solving particular problems, remember that the suitable sign among $\pm$ in (1) is to be chosen).

Problem 1. Objects A, B, and C are moving uniformly so that intersections of their position-time graphs form isosceles triangle (see Figure 1). If velocities of A and B are $v_A = -5.5$ m/s and $v_B = 2$ m/s, find velocity v_C of C.

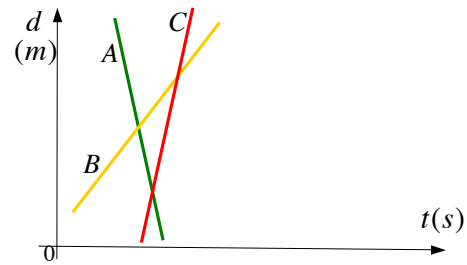


Figure 1.

Solution to problem 1. If the triangle formed is isosceles, then graph C is parallel to the line that bisects angle between A and B. Since velocity equals to the slope of position-time graph using (1) we get

$a = v_A + v_B = -3.5$, $b = 1 - v_A v_B = 12$, and

$$v_C = m_{bis} = \frac{a}{b - \sqrt{a^2 + b^2}} = \frac{-3.5}{12 - 12.5} = 7$$

m/s.

Problem 2. Cables AS and BS of the suspension bridge, shown in Figure 2, have slopes $m_1 = 1$ and $m_2 = 7$ respectively. Determine the slope m_0 of cable MS, which bisects angle between cables AS and BS.

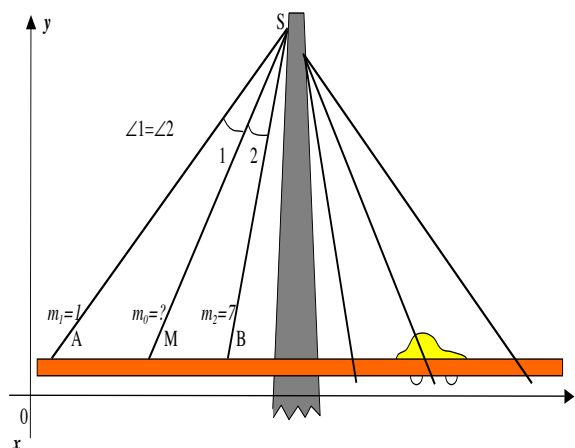


Figure 2.

Solution to problem 2. From (1) we get

$$a = m_1 + m_2 = 8, \quad b = 1 - m_1 m_2 = -6, \quad \text{and} \\ m_0 = m_{bis} = \frac{a}{b + \sqrt{a^2 + b^2}} = \frac{8}{-6 + 10} = 2.$$

Problem 3. A rhombus is drawn on a coordinate plane so that its sides have the slopes $m_1 = 2$ and $m_2 = 38$. Determine the slopes of the rhombus' diagonals.

Solution to problem 3. Diagonals of rhombus bisect its angles. Hence from (1) we get $a = m_1 + m_2 = 40$, $b = 1 - m_1 m_2 = -75$, and diagonals' slopes are

$$m_{bis} = \frac{a}{b \pm \sqrt{a^2 + b^2}} = \frac{40}{-75 \pm 85} = 4, -\frac{1}{4}.$$

Problem 4. When a wind turbine is in the position such that axes of blades I and II have slopes $m_1 = 3\sqrt{3}$ and $m_2 = -\sqrt{3}/2$ respectively (see Figure 4). What is the slope m_3 of its third blade? (Give an exact answer.)

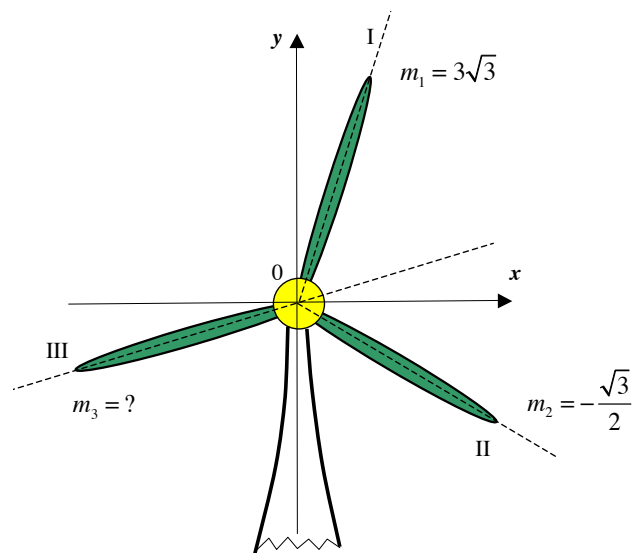


Figure 4.

Solution to problem 4. It is obvious that a third blade's axis bisects the angle between an axis of blade I and an axis of blade II. Thus using (1) we find

$$a = m_1 + m_2 = \frac{5\sqrt{3}}{2}, \quad b = 1 - m_1 m_2 = \frac{11}{2},$$

and

$$m_3 = m_{bis} = \frac{a}{b + \sqrt{a^2 + b^2}} = \frac{5\sqrt{3}}{11 + 14} = \frac{\sqrt{3}}{5}.$$

Problem 5. An incident ray (I), reflected ray (R), and mirror (M) are drawn on a coordinate plane, as shown on Figure 5. If an incident ray has a slope $m_1 = -1$, and reflected one has a slope $m_2 = 7$, what is the slope of a mirror

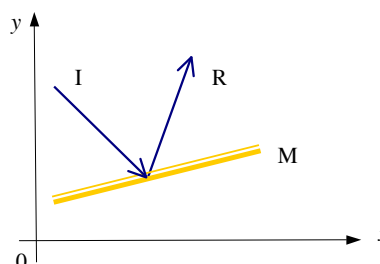


Figure 5.

Solution to problem 5. It is clear that if lines of rays I and R are continued beyond the mirror, the mirror's line

bisects angle between them. Consequently, applying (1) we obtain $a = m_1 + m_2 = 6$, $b = 1 - m_1 m_2 = 8$, and the slope of the mirror is $m_{bis} = \frac{1}{3}$.

Problem 6. An angled glass roof of a greenhouse is going to be supported by steel construction in form of equal isosceles triangles with aligned parallel bases (see Figure 6). Engineers' calculations of loads involve angles and slopes. If the slopes of segments AB and BC are $m_1 = \frac{1}{3}$ and $m_2 = -2$ respectively, what is the slope of the roof? (Give an exact answer.)

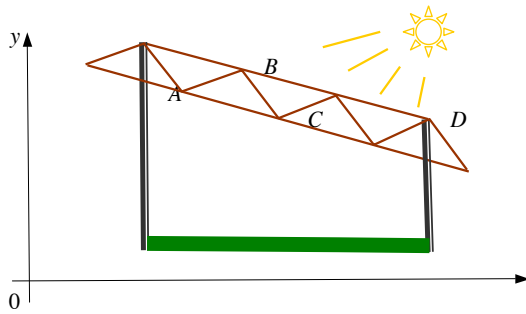


Figure 6.

Solution to problem 6. Since the triangles are isosceles, it is obvious that, if lines of segments AB and BC are continued beyond the construction, then a roof's line BD bisects an angle between them. Thus, using (1) we get $a = m_1 + m_2 = -\frac{5}{3}$, $b = 1 - m_1 m_2 = \frac{5}{3}$, and the exact answer for the question is $m_{bis} = \frac{a}{b + \sqrt{a^2 + b^2}} = \frac{-1}{1 + \sqrt{2}} = 1 - \sqrt{2}$.

Problem 7. A jet is turning gradually along a big horizontal circle in air so that its fuselage axis remains horizontal, and its wings axes l_1 and l_2 keep slopes $m_1 = -1$ and $m_2 = \frac{1}{7}$ respectively (as

shown in Figure 7). Determine the slope of the axis l_0 of jet's tail unit during this turn.

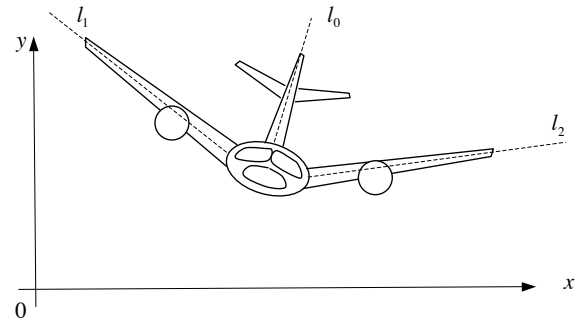


Figure 7.

Solution to problem 7. Since l_0 bisects an angle between l_1 and l_2 , using (1) we get $a = m_1 + m_2 = -\frac{6}{7}$, $b = 1 - m_1 m_2 = \frac{8}{7}$, and the slope of l_0 is $m_0 = m_{bis} = \frac{a}{b - \sqrt{a^2 + b^2}} = 3$.

Additional Examples (which may be used as exercises)

Problem 8. Upper and lower sides of a concrete mixer l_1 and l_2 (as shown in Figure 8) have slopes $m_1 = 1$ and $m_2 = 1/3$ respectively. Determine the slope of its axis of rotation (or axis of symmetry) l_0 . (Give an exact answer.)

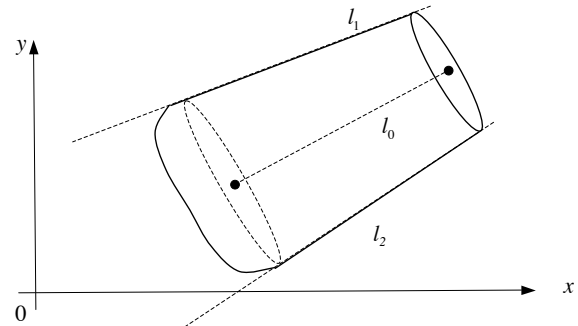


Figure 8.

Problem 9. Three objects A, B, and C are sliding uniformly along planes that have angles α , β , and γ with a horizontal plane, such that $\beta = \frac{\alpha + \gamma}{2}$ (see Figure 9). The coefficients of friction for planes A and C are $\mu_1 = \frac{1}{3}$ and $\mu_3 = \frac{1}{2}$ respectively. Find coefficient of friction μ_2 for plane B. (Give an exact answer.)

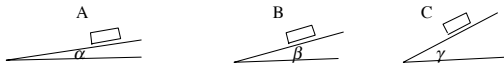


Figure 9.

Problem 10. Prove that $\tan 2\gamma = \frac{a}{b + \sqrt{a^2 + b^2}}$, where $a = \tan \gamma + \tan 3\gamma$, $b = 1 - \tan \gamma \tan 3\gamma$.

Problem 11. Graphs of linear functions (I, II, and III) are drawn (see Figure 11) using following tables:

Function I		Function II	
x	y	x	y
1	3	1	117
2	6	2	234
3	9	3	351

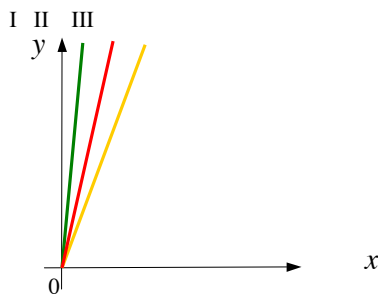


Figure 11.

If a graph II bisects an angle between graphs I and III, which table out of

tables A-D shown below represents graph II?

A		B		C		D	
x	y	x	y	x	y	x	y
1	6	1	12	1	30	1	60
2	12	2	24	2	60	2	120
3	18	3	36	3	90	3	180

Problem 12. An isosceles trapezoid is drawn on a coordinate plane so that its congruent sides have the slopes $m_1 = \frac{3}{4}$

and $m_2 = -\frac{5}{12}$. Determine the slope of its bases.

Solutions to the Additional Examples

8. $\frac{\sqrt{5}-1}{2}$; 9. $\sqrt{2}-1$; 10. Hint: show that if $m_1 = \tan \gamma$ and $m_2 = \tan 3\gamma$, then $m_{bis} = \tan 2\gamma$; 11. A; 12. $\frac{1}{8}$

It is clear that without formula (1) these types of problems are practically inaccessible for high school students. These are just a very few examples where (1) does work. A complete list of such examples is actually endless because an angle bisector is tied to symmetry and symmetry is a fundamental property of the nature. The examples discussed in this article represent a wide variety, which, I will argue, sufficiently justifies the addition of formula (1) to mathematics lessons in high school. I am confident that formal curriculum research will confirm the assertion.

Now it is time for teachers to demonstrate curiosity. Their question *should* be how to show that the statement (1) is true? Certainly, without

a proof this article is not completed. One possible way of proof is the following: Consider angles α , β , and γ that lines l_1 , l_2 , and q_1 , respectively, form with positive direction of x -axis (see Figure 1). Then $m_1 = \tan \alpha$, $m_2 = \tan \beta$, $m_{bis_1} = \tan \gamma$. Since q_1 is an angle bisector, it is obvious that $\gamma = \frac{1}{2}(\alpha + \beta)$.

Hence, for the slope m_{bis_1} of q_1 we get

$$\begin{aligned} m_{bis_1} &= \tan \gamma \\ &= \tan \left(\frac{1}{2} (\tan^{-1} m_1 + \tan^{-1} m_2) \right) \\ &= \tan \left(\frac{1}{2} \left(2 \tan^{-1} \frac{m_1 + m_2}{1 - m_1 m_2 + \sqrt{(1 + m_1^2)(1 + m_2^2)}} \right) \right) \\ &= \frac{m_1 + m_2}{1 - m_1 m_2 + \sqrt{(1 + m_1^2)(1 + m_2^2)}} = \frac{a}{b + \sqrt{a^2 + b^2}} \end{aligned}$$

where $a = m_1 + m_2$, $b = 1 - m_1 m_2$.

(Note, that identity (Akulov, 1999)

$$\begin{aligned} \tan^{-1} m_1 + \tan^{-1} m_2 \\ &= 2 \tan^{-1} \frac{m_1 + m_2}{1 - m_1 m_2 + \sqrt{(1 + m_1^2)(1 + m_2^2)}} \end{aligned}$$

is used for the proof, and its existence was suggested by the proof (Akulov, 1998) of

$$\sin^{-1} r + \sin^{-1} t = 2 \sin^{-1} \frac{1}{2} (\sqrt{u+v} - \sqrt{u-v})$$

, where $u = 1 + rt$, $v = r + t$

Second angle bisector q_2 is perpendicular to q_1 , and therefore its slope

$$m_{bis_2} = -\frac{1}{m_{bis_1}} = \frac{a}{b - \sqrt{a^2 + b^2}}. \text{ Q.E.D.}$$

It is useful to note, that the slope of the angle bisector relationship generalizes two separate items of the curriculum within one! Namely, if formula (1) is introduced, then the slope of parallel line and the slope of perpendicular line are automatically

introduced too because (1) contains them both. Indeed, if the special case when $m_1 = m_2 = m$ is applied, then m_{bis} immediately derives expressions for both, the slope m of parallel and slope $-\frac{1}{m}$ of perpendicular lines (see Figures B and C).

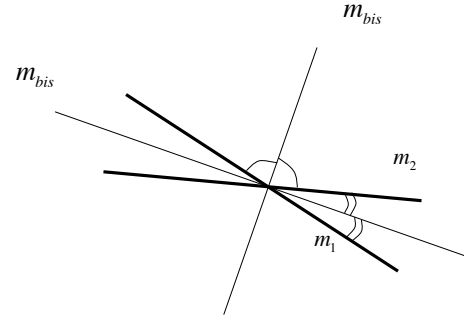


Figure B.

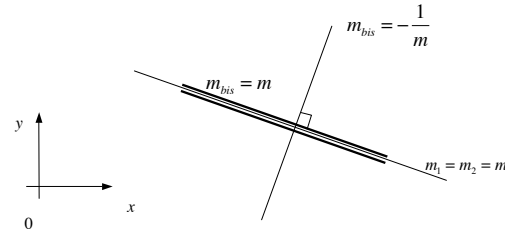


Figure C.

Therefore, I think, relationship (1) itself (certainly without formal proof, since the proof might be not accessible for most students) is an appropriate and relevant item for the high school mathematics curriculum today.

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SUM CONFERENCE POSTER

The Saskatchewan Mathematics Teachers' Society presents...

SUM: has changed will change is changing may 7-8, 2010

Featured presenters:

Kanwal Neel
Peter Liljedahl
Cathy Fosnot

Who: K-12 mathematics teachers
When: May 7-8, 2010
Where: Education Building,
University of Saskatchewan
Cost*: \$55 (SMTS members)
\$75 (non-members, includes
2-year SMTS membership)
*includes lunch on Friday

Did you ever think that SUM-day you would go to SUM conference...?
The annual SMTS Saskatchewan Understands Math Conference is for
K-12 math teachers interested in the new curriculum, incorporating
technology into their teaching, number sense, and problem solving.

<http://smts.ca>
for details and registration.
Deadline to register is April 30.

Sponsors:



SUM CONFERENCE SCHEDULE

Friday, May 7th

9:00-10:00	Keynote Address: Kanwal Neel Being the Change in Learning to Teach and Teaching to Learn.			
10:00-10:30	COFFEE			
10:30-12:30	K-3 Liliane Gauthier Big Ideas for Little People: Important things about Patterns	4-6 Peter Liljedahl Numeracy Tasks for Grades 4-6	6-9 Kanwal Neel Addressing Diversity in the Mathematics Classroom with Cultural Artifacts	10-12 Egan Chernoff Teaching and Learning Combinatorics in the Senior High School Mathematics Classroom with No Calculator and No Formulas: No Way!
12:30-1:15	LUNCH			
1:15-2:15	K-2	Trish Reeve	Developing Number Sense K-2	
	3-5	Tracey Young	I LOVE having students develop their own understanding in my Math class but.....	
	6-9	Harley Weston, Andrea Rogers, Jessica Wesauate	Aboriginal Perspectives	
	9-12	Brian Crawley	Mathematics Lessons Using Learner-Generated Examples (LGEs)	
	4-8	Christina Fonstad	Mrs. Dress Up and the Math Tickle Trunk	
	7-12	Gary Davis	Using Examview to Create Assessments and Extra Practice	
	COFFEE			
2:30-3:30	Keynote Address: Peter Liljedahl Student Behaviour as Sensible Reflections of our Teaching			

Saturday, May 8th

9:00-10:45	K-5 Cathy Fosnot Using Strings of Related Problems (Minilessons) to Promote Numeracy	7-9 Peter Liljedahl Numeracy Tasks for Grades 7-9	6-9 Kanwal Neel Addressing Diversity in the Mathematics Classroom with Cultural Artifacts	10-12 Egan Chernoff Non-Combinatorial Probability Problems in the Senior High School Mathematics Classroom.
10:45-11:00	COFFEE			
11:00-12:00	K-5	Cathy Fosnot & Craig Featherstone	Using Strings of Related Problems (Minilessons) to Promote Numeracy	
	6-10	Karen Campbell	Middle Years Geometry	
	6-10	Stephen Vincent	TBA	
	10-12	Gary Davis	Using Math XL	
	10-12	Gregory Akulov	The Slope of the Angle Bisector Relationship in Problem Solving and Curriculum enrichment	
12:00-1:00	Keynote Address: Cathy Fosnot Developmentally-appropriate Practice vs. a Practice that Develops Young Mathematicians: the Case of Multiplication			

SUM CONFERENCE SESSION DESCRIPTIONS

Friday, May 7th

9:00am

Being the Change in Learning to Teach and Teaching to Learn.

***Kanwal Neel* (Keynote address)**

It has been over thirty years since I started my career as a mathematics educator. Much has changed; is changing; will change in the teaching and learning of mathematics. In this presentation I will share some of the lessons I have learned and the changes I have adapted to become a more effective teacher.

10:00am

(K-3) Big Ideas for Little People: Important things about Patterns

Liliane Gauthier

Through hands-on activities, children's literature and "what if" questions, this presentation will address the fundamental ideas about repeating patterns to help children build a solid foundation of understanding and experiences to prepare for more sophisticated work in algebraic thinking

(4-6) Numeracy Tasks for Grades 4-6

Peter Liljedahl

With the latest curriculum revisions *numeracy* has taken on a more prominent role within the 4-6 curriculum. But exactly what is this thing that we call numeracy? In this workshop we will examine this notion of numeracy and how it is different than mathematics. Participants will engage in, and leave with, a number of grade 4-6 numeracy tasks for use in meeting the curriculum's demand for attainment of this elusive, yet important, learning outcome.

(6-9) Addressing Diversity in the Mathematics Classroom with Cultural Artifacts

Kanwal Neel

Participants will be introduced to the use of cultural artifacts and creative stories, enabling one to teach and assess mathematics in an imaginative and creative way that both respects diversity and extends the understanding to a personal context. A special project, which showcases the numeracy practices at the Squamish Lil'wat Cultural Centre (SLCC), BC, will be shared along with student samples and a hand out outlining the project.

(10-12) Teaching and Learning Combinatorics in the Senior High School Mathematics Classroom with No Calculator and No Formulas: No Way!

Egan Chernoff

The purpose of this session is to provide teachers with an opportunity to experience the reform approach to the teaching and learning combinatorics. Engaging with combinatorially rich tasks, embracing the notion of multiple selves, and suspending our reality will preface a discussion on the tensions between theoretical and practical issues found in both reform and traditional classrooms.

1:15

(K-2) Developing Number Sense K-2

Trish Reeve

This workshop will focus on instructional strategies and activities for Pre K – grade 2 students to understand numbers, be able to represent them in different ways, and explore relationships among numbers.

(3-5) I LOVE having students develop their own understanding in my Math class but.....

Tracey Young

Conversations and problem solving around issues and difficulties in today's Mathematics classroom.

(6-9) Aboriginal Perspectives

Harley Weston, Andrea Rogers, and Jessica Wesauate

The Aboriginal Perspectives website (<http://AboriginalPerspectives.uregina.ca/>) has video clips of traditional Aboriginal activities and interviews with Aboriginal people along with associated lesson ideas across the curriculum. The presentation will consist of a brief introduction, a tour of the site, and an indication of future plans.

(9-12) Mathematics Lessons Using Learner-Generated Examples (LGEs)

Brian Crawley

Examples play a big role in mathematics education, but often the source of these examples is a teacher or a textbook. Learner-generated examples (LGEs) is a teaching and learning strategy in which the teacher invites students to create their own examples as they try to make sense of concepts. In this session, participants will experience the process of generating examples so that they might feel comfortable trying this strategy in their own classrooms. As well, they will be provided with some practical lesson plans that are aligned with the new WNCB curriculum.

(4-8) Mrs. Dress Up and the Math Tickle Trunk

Christina Fonstad

In this session you will be introduced to *The Math Tickle Trunk*. The speaker initially used *The Math Tickle Trunk* as a resource to meet the diverse needs of students in her upper elementary and middle years mathematics classes. She also found that it was a motivational and classroom management tool. But can the Math Tickle Trunk be more?

(7-12) Using Examview to Create Assessments and Extra Practice

Gary Davis

2:30

Student Behaviour as Sensible Reflections of our Teaching:

Peter Liljedahl (Keynote address)

As teachers we try to be consistent in our delivery of a set curriculum. We do so for the benefit of the consumers of our teaching – the students. Ironically, the students in this endeavor are varied, inconsistent, and sometimes unpredictable. But what if we saw their

behaviours as sensible reflections of our teaching? In this talk I will use this untenable stance to examine mathematics education writ large and to present the questions critical to the development of our teaching.

Saturday, May 8th

9:00am

(K-5) Using Strings of Related Problems (Minilessons) to Promote Numeracy

Cathy Fosnot

In this workshop, participants will first be asked to be learners. Strings crafted with related problems will be used to develop a repertoire of mental arithmetic strategies. The structure and choice of problems will subsequently be analyzed and video of elementary students, grades K-6, will be used to examine the practice with children. Emphasis will be placed on the role of representation with models such as the open number line, the arithmetic rack, the ratio table, clocks, and coins.

(7-9) Numeracy Tasks for Grades 7-9

Peter Liljedahl

With the latest curriculum revisions *numeracy* has taken on a more prominent role within the 7-9 curriculum. But exactly what is this thing that we call numeracy? In this workshop we will examine this notion of numeracy and how it is different than mathematics. Participants will engage in, and leave with, a number of grade 7-9 numeracy tasks for use in meeting the curriculum's demand for attainment of this elusive, yet important, learning outcome.

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Participants will be introduced to the use of cultural artifacts and creative stories, enabling one to teach and assess mathematics in an imaginative and creative way that both respects diversity and extends the understanding to a personal context. A special project, which showcases the numeracy practices at the Squamish Lil'wat Cultural Centre (SLCC), BC, will be shared along with student samples and a hand out outlining the project.

(10-12) Non-combinatorial Probability Problems in the Senior High School Mathematics Classroom:

Egan Chernoff

The purpose of this session is to provide teachers with an opportunity to experience the reform approach to the teaching and learning probability, which does not involve combinatorics. Engaging with probabilistically rich tasks, embracing the notion of multiple selves, and suspending our reality will preface a discussion on the tensions between theoretical and practical issues found in both Reform and Traditional classrooms.

11:00am

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(6-10) Middle years geometry

Karen Campbell

(6-10) TBA

Steve Vincent

(10-12) Using Math XL

Gary Davis

(10-12) The Slope of the Angle Bisector Relationship in Problem Solving and Curriculum enrichment

Gregory Akulov

Years ago, to learn how to find the slope of the angle bisector a student had to take a university Math class. However today the term the “slope of the angle bisector” is ready to be included in high school mathematics curriculum because of concise and suitable relationship found in Saskatchewan (2008) and first published in Manitoba (2009). The slope of the angle bisector objective is directly and tightly related to mathematics curriculum (angle bisector, parallel and perpendicular lines, slope relationships, symmetry, axis of symmetry, graph of linear function, rate of change), to calculus curriculum (tangent line, derivative, instantaneous rate of change), and to science curriculum (position-time graph, velocity, uniform motion, motion with uniform acceleration, velocity-time graph, Ohm's law, resistance). This presentation includes brief history of discovery and perfect system of original applied examples.

12:00pm

Developmentally-appropriate Practice vs. a Practice that Develops Young Mathematicians: The Case of Multiplication

Cathy Fosnot (Keynote address)

Developmentally-appropriate practice usually is depicted by diagnosis of the stage of learning followed by a set of matching, prescribed “hands-on” activities. In this session, we will explore and contrast a practice that is designed to foster the development of young mathematicians by involving them in rich, open-ended but didactically-crafted, investigations. Young mathematicians even at an early age thrive on inquiring and defending their solutions in communities of discourse and reflection. The topic of multiplication and its development will be used as an example.

vinculum

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VOLUME 2, NUMBER 2 (OCTOBER 2010)

First Nations, Métis, & Inuit Content, Perspectives, and Ways of Knowing

Gale Russell
(Guest Co-Editor)

This special issue of *vinculum* will focus on First Nations, Métis and Inuit (FNMI) content, perspectives, and ways of knowing. With the ongoing renewal of Saskatchewan's mathematics curricula, indicators and outcomes have been included to help situate FNMI content, perspectives, and ways of knowing within the teaching and learning of mathematics. Teachers of Aboriginal and non-Aboriginal students alike are seeking knowledge and insights into how to authentically, meaningfully, and respectfully bring FNMI content, perspectives, and ways of knowing into their mathematics classrooms.

For this special issue, we invite contributions from different perspectives and contexts that focus on describing FNMI content, perspectives, and ways of knowing related to mathematics, the learning of mathematics, and the teaching of mathematics. Questions of interest include: how FNMI perspectives and mathematical ways of knowing are different from those of the traditional mathematics classroom, resources, and teaching and learning; what FNMI content can be meaningfully and respectfully brought into the mathematics classroom; what resources, including Elders and electronic resources, can be used to effectively bring FNMI content, perspectives, and ways of knowing into the mathematics classrooms; and what pedagogical approaches support effective and authentic merging of FNMI content, perspectives, and ways of knowing into the renewed mathematics curriculum. In addition, we welcome stories and studies related to the successful inclusion of FNMI content, perspectives, and ways of knowing in the mathematics classroom.

The Journal of the Saskatchewan Mathematics Teachers' Society, *vinculum*, is seeking **Articles** and **Conversations** for the upcoming October 2010 edition. Given the wide range of parties interested in teaching and learning mathematics, we invite submissions for considerations from any interested persons, but, as always we encourage Saskatchewan's teachers of mathematics to be our main contributors. All contributions must be submitted to glr127@mail.usask.ca by September 1, 2010 to be considered for inclusion in the October issue.



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